Work and Energy (4 pages; 21/11/20)

(1) Work and energy are two sides of the same coin: when a force does work on an object, the objects gains energy.

Some forces, such as friction, do negative work. An object is said to do work against friction.

(2) Work done by a force

This is defined to be:

Component of the force in the direction of motion

× distance moved

[Integration will be needed if the direction of motion changes.]

Notes

(i) This is just an extension of the definition $W = Fs$. A force can be broken down into two components: parallel and perpendicular to the direction of motion. As the distance moved perpendicular to the direction of motion is zero, only the parallel component contributes to Fs.

(ii) $W = Fs$ can be justified as follows, in the case of constant acceleration:

$v^2 - u^2 = 2as$

$\Rightarrow \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = (ma)s = Fs,$

so that $W = Fs$, given that the work done is equal to the kinetic energy gained (by the Work-Energy principle, discussed below).

(3) Types of Energy

Mechanical

Kinetic, potential (gravitational / elastic)

Non-mechanical

Heat, sound, light, electrical, chemical, nuclear

(4) Potential energy

In Mechanics, kinetic energy can be thought of as the present energy possessed by an object, whilst potential energy is future energy for which credit is being taken, on the basis that work has already been done to store up this potential energy, and the relevant force (gravity or the tension in an elastic rope, for example) can be relied on to do work equal to this potential energy, once the object is allowed to move.

Thus, in the case of gravitational potential energy, work is done against gravity to raise an object to a particular height. In the case of elastic potential energy, work is done to stretch an elastic rope, for example.

(5) Conservation of Energy

The principle of Conservation of Energy states that

$\mathrm{KE} + \mathrm{PE} = \text{constant},$

provided that there are no external forces, other than the forces giving rise to the potential energy (such as gravity or the tension in an elastic rope).

Forces that can give rise to potential energy are termed 'conservative'. Friction is an example of a non-conservative (or 'dissipative') force.

In situations where other forces exist (either where work is being done on an object, or where an object loses energy in 'work done against friction', for example), the principle of Conservation of Energy can be adapted to give:

Work done on an object = net gain in KE & PE (A)

or (for example):

Work done against friction = net loss of KE & PE (B)

[In many situations, a gain in KE will be accompanied by a loss of PE (or vice versa), in which case the RHS of (A) becomes:

Gain in KE – Loss in PE]

(6) Work-Energy principle

This is an alternative to adapting the principle of Conservation of Energy. It states that:

Total work done by all forces acting on an object

= increase in the object's KE

This total work should include any work done by gravity or other conservative forces, as no potential energy is being allowed for.

(7) Example: A cyclist of mass m rides downhill, doing work W against friction. His starting speed is u and his finishing speed is v. He moves through a vertical height of h.

(A) Adapting the principle of Conservation of Energy:

Work done against friction = Loss of PE – Gain in KE,

so that $W = mgh - \frac{1}{2}m(v^2 - u^2)$

(B) Work-Energy principle:

$-W + (mg)h = \frac{1}{2}m(v^2 - u^2)$

[If an object moves a distance d down a slope inclined at an angle θ to the horizontal, the component of gravity down the slope is

$mgsin\theta$, so that the work done by gravity is

$mgsin\theta . d = mg(dsin\theta) = (mg)h$,

where h is the change in height of the object.]