Work & Energy - Exercises (with Sol'ns)

(9 pages; 10/2/26)

(1) A car of mass 1 tonne starts to climb a hill at $20ms^{-1}$. The slope of the hill is a constant θ, where $sin\theta = \frac{1}{10}$. If the car is not accelerating (or braking) and there is a constant resistance to motion of $1000N$, find the speed of the car when it has gained a height of $5m$. Assume that $g = 10$.

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Solution

Method 1

By the Work-Energy principle,

Gain in KE = Work done by forces,

so that $\frac{1}{2}(1000)(v^2 - 20^2) = -1000g(5) - 1000\left(\frac{5}{sin\theta}\right)$

$\Rightarrow 500v^2 = 200000 - 50000 - 50000$

$\Rightarrow v^2 = 200 \Rightarrow v = 14.1\ ms^{-1}\ (3sf)$

Method 2

By Conservation of Energy,

Gain in PE = loss of KE − work done against resistance

$\Rightarrow 1000g(5) = \frac{1}{2}(1000)(20^2 - v^2) - 1000\left(\frac{5}{sin\theta}\right)$

which gives the same equation.

(2) A car of mass $1200kg$ pulls a trailer of mass $400kg$. There are resistances of $400N$ and $100N$ on the car and trailer, respectively.

(i) If an acceleration of $0.2ms^{-2}$ is possible when travelling at $20ms^{-1}$, find the maximum speed of the car.

(ii) If the trailer is connected to the car by means of a rope, what is the maximum deceleration that is possible?

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Solution

(i) Considering the car and trailer combined, if X is the driving force of the car's engine:

$N2L \Rightarrow X - (400 + 100) = (1200 + 400)(0.2)$

$\Rightarrow X = 820N$

The (maximum) power of the car's engine is therefore

$820(20) = 16400$

At maximum speed, the driving force is equal to the total resistance, $500N$, and the maximum speed, v_{max} is given by

$16400 = 500v_{max}$,

so that $v_{max} = 32.8ms^{-1}$

(ii) Considering the trailer,

$N2L \Rightarrow T - 100 = 400a$,

where T is the tension, and a is the acceleration

The rope cannot be in compression, so $T > 0$.

Hence $T = 400a + 100 > 0$

$\Rightarrow a > -0.25$

ie the maximum possible deceleration is $0.25ms^{-2}$

(3) A car of mass $1200kg$ starts to descend a slope at $10ms^{-1}$. The slope is at a constant angle θ to the horizontal, where $sin\theta = \frac{1}{10}$. If the car is not accelerating or braking, and there is a constant resistance to motion of $500N$, find the speed of the car when it has travelled $100m$. Assume that $g = 10ms^{-2}$.

(3) A car of mass $1200kg$ starts to descend a slope at $10ms^{-1}$. The slope is at a constant angle θ to the horizontal, where $sin\theta = \frac{1}{10}$. If the car is not accelerating or braking, and there is a constant resistance to motion of $500N$, find the speed of the car when it has travelled $100m$. Assume that $g = 10ms^{-2}$.

Solution

Method 1

By the Work-Energy principle,

Gain in KE = Work done by forces,

so that $\frac{1}{2}(1200)(v^2 - 10^2) = 1200g(100sin\theta) - 500(100)$

$\Rightarrow 600v^2 = 120000 - 50000 + 60000 = 130000$

$\Rightarrow v^2 = \frac{650}{3} \Rightarrow v = 14.7\ ms^{-1}\ (3sf)$

Method 2

By Conservation of Energy,

work done against resistance = loss of PE − gain in KE

$\Rightarrow 500(100) = 1200g(100sin\theta) - \frac{1}{2}(1200)(v^2 - 10^2)$,

which gives the same equation.

(4) A block of mass $5kg$ is initially ascending a slope at a speed of $2ms^{-1}$. The slope has a gradient of 0.75, and the only resistance to motion is a frictional force of $20N$.

(i) How far up the slope does the block travel?

(ii) What is the total time taken for the block to travel up the slope and return to its starting point?

(4) A block of mass $5kg$ is initially ascending a slope at a speed of $2ms^{-1}$. The slope has a gradient of 0.75, and the only resistance to motion is a frictional force of $20N$.

(i) How far up the slope does the block travel?

(ii) What is the total time taken for the block to travel up the slope and return to its starting point?

Solution

(i) By conservation of energy,

Work done against friction = loss of KE − gain in PE

So $20d = \frac{1}{2}(5)(2^2) - (5)(9.8)dsin\theta$,

where d is the distance moved, and $tan\theta = 0.75 = \frac{3}{4}$,

so that $sin\theta = \frac{3}{5}$ (from the Pythagorean triple 3,4,5)

Thus $20d = 10 - \frac{147d}{5}$,

$\Rightarrow 100d = 50 - 147d \Rightarrow d = \frac{50}{247} = 0.202m$ or $20.2cm$ (3sf)

(ii) Up the slope, by N2L:

$-5gsin\theta - 20 = 5a$, where a is the acceleration up the slope

So $a = -(9.8)\left(\frac{3}{5}\right) - 4 = -\frac{247}{25} = -9.88\ ms^{-2}$

If t is the time taken to go up the slope,

$'v = u + at' \Rightarrow 0 = 2 + \left(-\frac{247}{25}\right)t$

$\Rightarrow t = \frac{50}{247} = 0.20243s$

Down the slope, by N2L:

$5gsin\theta - 20 = 5a'$, where a' is the acceleration down the slope

So $a' = (9.8)\left(\frac{3}{5}\right) - 4 = \frac{47}{25} = \ 1.88\ ms^{-2}$

From (i), $d = \frac{50}{247}$

If t' is the time taken to go down the slope,

$'s = ut + \frac{1}{2}a(t')^{2'} \Rightarrow \frac{50}{247} = 0 + \frac{1}{2}(\frac{47}{25})(t')^2$

$\Rightarrow (t')^2 = \frac{2500}{11609} \Rightarrow t' = 0.46406$

So total time is $\ 0.20243 + 0.46406 = 0.66649 = 0.666s$ (3sf)