Volumes of Revolution, Surface Area & Arc length

Exercises (2 pages; 27/3/26)

(1) The region between the line $y = 6 - 2x$ and the curve $y = \frac{4}{x}$ is rotated about the y-axis through 360°. Find the exact volume generated.

(2) The region between the parabola $y^2 = 4x$, the x-axis and the line $x = 1$ is rotated about the x-axis through 360°.

(i) Find the exact volume generated:

(a) by integrating with respect to x

(b) by integrating with respect to the parameter t, where $x = t^2$ and $y = 2t$

(ii) Use the mean value of the function to carry out a rough check on your answer in (i).

(iii) Find the curved surface area associated with the volume generated in (i):

(a) by integrating with respect to x

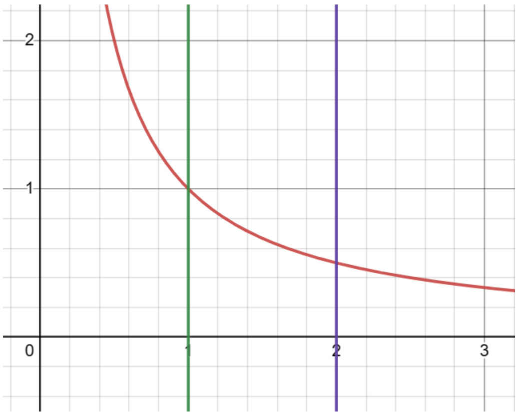
(b) by integrating with respect to y

(c) by integrating with respect to t

(3) The curve C has equation $y = \frac{1}{3}x^3 + \frac{1}{4x}$. The points A and B on C have x coordinates 1 and 2, respectively. Find the length of the arc from A to B.

(4) Use integration with respect to a suitable parameter to show that the surface area of a sphere of radius r is $4\pi r^2$.

(5) The region bounded by the curve $y = \frac{1}{x}$, the lines $x = 1, x = 2$, and the x-axis is rotated about the y-axis through 360°. Find the volume generated.



(6) The region between the curve $y = 4 - x^2$, the coordinate axes, and the line $x = 1$ is rotated about the y-axis. Find the volume of the solid generated.

(7) Let V_x be the volume of revolution when the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is rotated about the x-axis, and V_y the volume when it is rotated about the y-axis. Show that $\frac{V_x}{V_y} = \frac{b}{a}$.