Vectors - Introduction (6 pages; 4/8/18)

[These notes only cover vectors in 2 dimensions.]

(1) A vector can be used to represent anything that has a magnitude (ie size) and a direction.

Examples

(i) A 2-dimensional displacement, which can be thought of as a navigational instruction; eg "travel North-East for 300m".

(ii) A velocity of $5ms^{-1}$ at an angle of 20^0 to the horizontal (in a vertical plane).

(iii) The acceleration of an object.

(iv) A force on an object.

Notes

(a) Items such as temperature, mass or speed have no direction associated with them, and are referred to as scalars.

(b) Note that, subject to (c) below, it is not specified where a vector acts. Thus, two objects moving parallel to each other may have the same velocity vector. In Mechanics it is usually essential to know where a force acts (or the line along which it acts), but this information has to be provided in addition to its magnitude and direction.

(c) An exception to (b) is the position vector, which is always anchored at the Origin. It is not really a true vector; but rather another way of representing the coordinates of an object. However, ordinary vectors (sometimes referred to as free vectors) are often drawn starting from the Origin, for convenience.

(d) In the diagram below, the 3 vectors $\underline{a}$ are all equivalent (as they have the same magnitude and direction), though the position vector associated with the point A has to be drawn from the Origin.
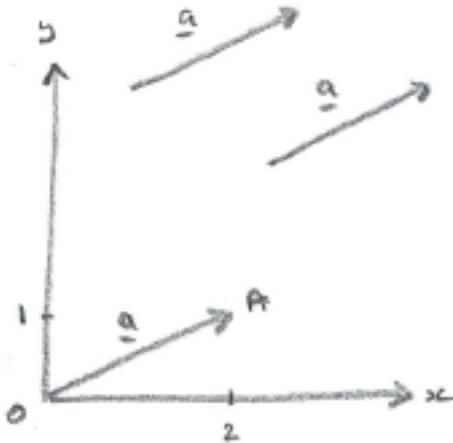
(2) Notation

The notation $\underline{a}$ is used when written by hand. In print, it appears in bold: $\boldsymbol{a}$

The notation $\overrightarrow{OA}$, $\overrightarrow{AB}$ is also used to represent the vectors joining the points O and A, and A and B, respectively. $\overrightarrow{OA}$, $\overrightarrow{OB}$ etc are commonly written as $\underline{a}$ and $\underline{b}$ (though, strictly speaking, this notation should be defined).

Unit vectors $\underline{i}$ and $\underline{j}$ are defined as having magnitude 1, and being in the directions of the x and y axes, respectively.

Any vector can be written (in one way only) in the form $a\underline{i} + b\underline{j}$, or alternatively in 'column vector' form as $\begin{pmatrix} a \\ b \end{pmatrix}$. Thus, in the diagram above, $\underline{a} = 2\underline{i} + \underline{j}$ or $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

(3) Calculation of magnitude and direction

In the diagram above, the magnitude of $\underline{a}$, denoted by $|\underline{a}|$ is found using Pythagoras' theorem: $|\underline{a}| = \sqrt{2^2 + 1^2} = \sqrt{5}$

Note: Depending on whether the vector represented eg a velocity or a force, appropriate units would be needed.

The direction is usually the angle measured from the horizontal (though this should be specified); in this case it is $tan^{-1}\left(\frac{1}{2}\right) = 26.6°$ (3sf)

(4) Multiplication of a vector by a scalar

The magnitude is changed accordingly, but the direction remains the same, except when multiplying by a negative number, in which case the direction is reversed. This can be seen from the column vector representation; eg $-4\begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}-8\\-12\end{pmatrix}$
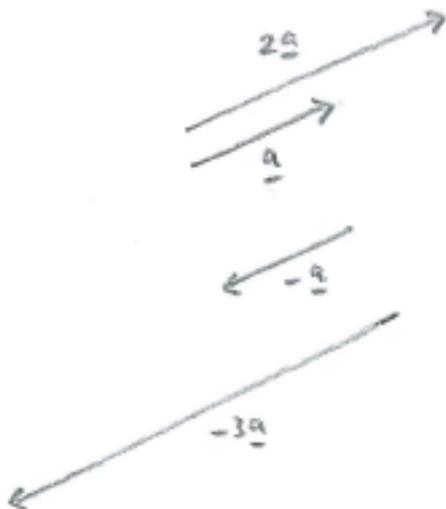
(5) Addition of vectors

Eg a navigational instruction to go along 2 and up 1, followed by another instruction to go along 1 and up 3 is equivalent to a single instruction to go along 3 and up 4 (see diagram below).
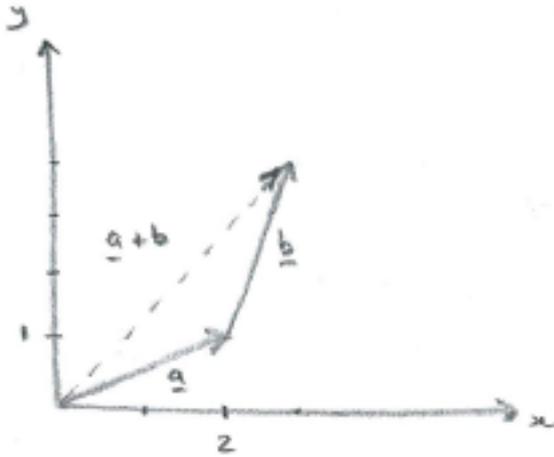
In terms of column vectors: $\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

The above diagram shows the application of the 'triangle rule'. If both vectors are drawn from the Origin, then the resultant (ie their sum) is the diagonal of the parallelogram (see diagram below). This is obtained from either $\underline{a} + \underline{b}$ or $\underline{b} + \underline{a}$ (according to which way we go round the parallelogram).
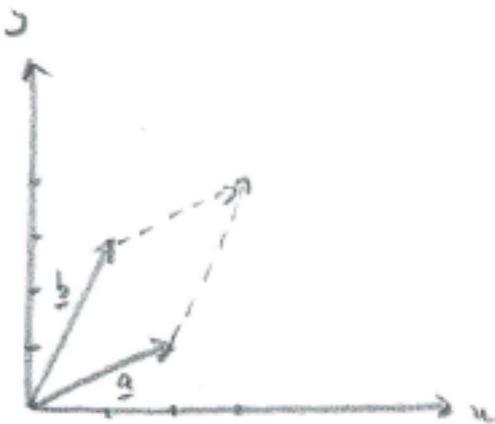
(6) Subtraction of vectors
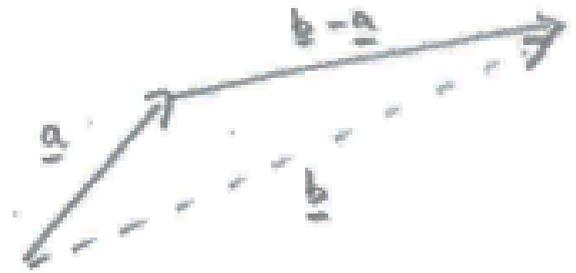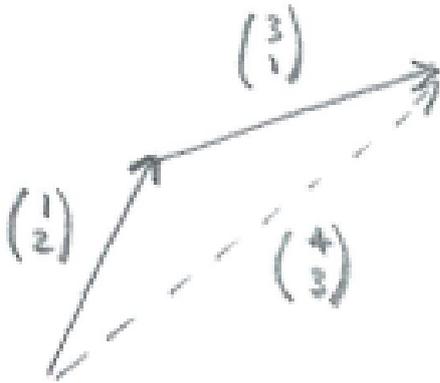
$$\begin{pmatrix}1\\2\end{pmatrix} + \begin{pmatrix}3\\1\end{pmatrix} = \begin{pmatrix}4\\3\end{pmatrix}$$

$$\begin{pmatrix}4\\3\end{pmatrix} - \begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}3\\1\end{pmatrix}$$

In the above diagrams, the dashed vector is the resultant of the other two vectors, and this enables us to find the difference between two vectors, as shown in the right-hand diagram.

As another way of looking at this, a common vector device is to find two routes between two given points. Thus, referring to the diagrams below, $\underline{a} + \underline{x} = \underline{b}$, so that $\underline{x} = \underline{b} - \underline{a}$
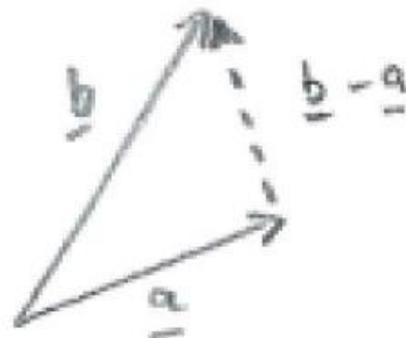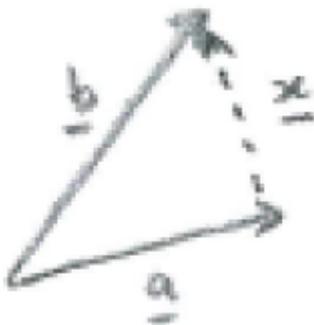
(7) Components of a vector

Referring to the diagram,

the right-angled triangle with hypotenuse v produces

$cos30° = \frac{v_x}{v}$ & $sin30° = \frac{v_y}{v}$

so that:

$v_x = vcos30° = \frac{v\sqrt{3}}{2}$

$v_y = vsin30° = \frac{v}{2}$