Vectors Exercises - Part 1 (5 pages; 19/5/20)

Key to difficulty:

* easier

** moderate

*** harder

(1*) Vector equation of line

Given that the line $\underline{r} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ can also be written as $\begin{pmatrix} 0 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 2 \end{pmatrix}$, find μ in terms of λ

(2*) Vector equation of line

Find a vector equation of the line that passes through the point (1,2) and is perpendicular to the line $\underline{r} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

(3**) Scalar product

Show that if $|\underline{a} - \underline{b}| = |\underline{a} + \underline{b}|$, then $\underline{a}$ & $\underline{b}$ are perpendicular.

(4**) Planes

Find the cartesian form of the plane

$$\underline{r} = \begin{pmatrix} 0 \\ -2 \\ -1 \end{pmatrix} + s \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} + t \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

(5*) Lines and planes**

Find the line that is the reflection of the line $\frac{x-2}{3}=\frac{y}{4}=\frac{z+1}{1}$ in the plane $x-2y+z=4$

(6) Lines and planes**

(i)(a) Find the acute angle between the line $\boldsymbol{\frac{x}{2}=\frac{y+1}{-3}=\frac{z-2}{1}}$ and the plane $\boldsymbol{x+y-2z=5}$

(b) Show that the same angle is obtained if the line is written in the form

$\boldsymbol{\frac{x}{-2}=\frac{y+1}{3}=\frac{z-2}{-1}}$ (ie without rearranging into the form in (a))

(ii)(a) Find the acute angle between the planes $\boldsymbol{x+4y-3z=7}$ and

$\boldsymbol{x-y+4z=2}$

(b) Find the acute angle between the planes $\boldsymbol{x+4y-3z=7}$ and

$\boldsymbol{-x+y-4z=2}$ (again, without rearranging the equation)

(7*) Lines and planes**

Find the line that is the reflection of the line $\boldsymbol{\frac{x-2}{3}=\frac{y}{4}=\frac{z+1}{1}}$ in the plane $\boldsymbol{x-2y+z=4}$

(8*) Lines**

Find the distance between the lines $\boldsymbol{\frac{x+1}{1}=\frac{y+2}{2}; z=4}$ and

$\frac{x+3}{1}=\frac{y-6}{2}$; $z=7$, leaving your answer in exact form.

(9*) Lines**

(i) Show the lines $\frac{x-1}{2}=\frac{y+3}{5}=\frac{z-2}{3}$ and $\frac{x}{1}=\frac{y-4}{2}=\frac{z+1}{2}$ are skew.

(ii) Find the shortest distance between the lines and identify the points on the lines at which this shortest distance occurs.

(10) Lines**

Given that $A=\begin{pmatrix}1\\2\\3\end{pmatrix}$, $B=\begin{pmatrix}-4\\3\\1\end{pmatrix}$, $C=\begin{pmatrix}0\\1\\2\end{pmatrix}$, $D=\begin{pmatrix}p\\4\\-4\end{pmatrix}$

(i) Write down the equations of the lines AB and CD (both extended)

(ii) Find $\overrightarrow{AB}\times\overrightarrow{CD}$

(iii) For what value of p are the lines AB and CD parallel? (2 methods)

(11) Planes**

Find the plane containing the points
$(2,-1,4)$, $(-3,4,2)$ and $(1,0,5)$, in Cartesian form

(12*) Lines and planes**

Find the reflection of the line $\frac{x-2}{3}=\frac{y+4}{1}$; $z=3$ in the plane $y=4$

(13*) Problem**

Use vectors to prove that the mid-points of the sides of any quadrilateral form the vertices of a parallelogram.

(14) Distance from point to plane**

(i) Find the intersection of the line $\underline{r} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$ and the plane $3x + y + 4z = 77$

(ii) Find the shortest distance from the point $\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$ to the plane

$3x + y + 4z = 77$

(15*) Distance from point to plane**

Show that the shortest distance from the point $\underline{p}$ to the plane $\underline{r}.\underline{n} = d$ is $\frac{|d - \underline{p}.\underline{n}|}{|\underline{n}|}$

(16) Distance from point to plane**

(i) Given that the shortest distance from the point $\underline{p}$ to the plane $\underline{r}.\underline{n} = d$ is $\frac{|d - \underline{p}.\underline{n}|}{|\underline{n}|}$, what is the significance of $\frac{d}{|\underline{n}|}$ if $d > 0$?

(ii) Find the equation of the plane that is parallel to $\underline{r}.\underline{n} = d$ and contains the point $\underline{p}$.

(iii) Hence deduce the formula for the shortest distance from the point $\underline{p}$ to the plane $\underline{r}.\underline{n} = d$

(17**) Volume of tetrahedron

Find the volume of the tetrahedron with corners
$(2, 1, 3), (-1, 5, 0), (4, 4, 7), (8, 2, 2)$