Vectors Exercises - Harder (5 pages; 21/1/21)

(1) Lines and planes

Find the line that is the reflection of the line $\frac{x-2}{3}=\frac{y}{4}=\frac{z+1}{1}$ in the plane $x-2y+z=4$

(2) Lines and planes

Find the line that is the reflection of the line $\frac{x-2}{3}=\frac{y}{4}=\frac{z+1}{1}$ in the plane $x-2y+z=4$

(3) Lines

Find the distance between the lines $\frac{x+1}{1}=\frac{y+2}{2}$; $z=4$ and $\frac{x+3}{1}=\frac{y-6}{2}$; $z=7$, leaving your answer in exact form.

(4) Lines

(i) Show the lines $\frac{x-1}{2}=\frac{y+3}{5}=\frac{z-2}{3}$ and $\frac{x}{1}=\frac{y-4}{2}=\frac{z+1}{2}$ are skew.

(ii) Find the shortest distance between the lines and identify the points on the lines at which this shortest distance occurs.

(5) Lines and planes

Find the reflection of the line $\frac{x-2}{3} = \frac{y+4}{1}; z = 3$ in the plane $y = 4$

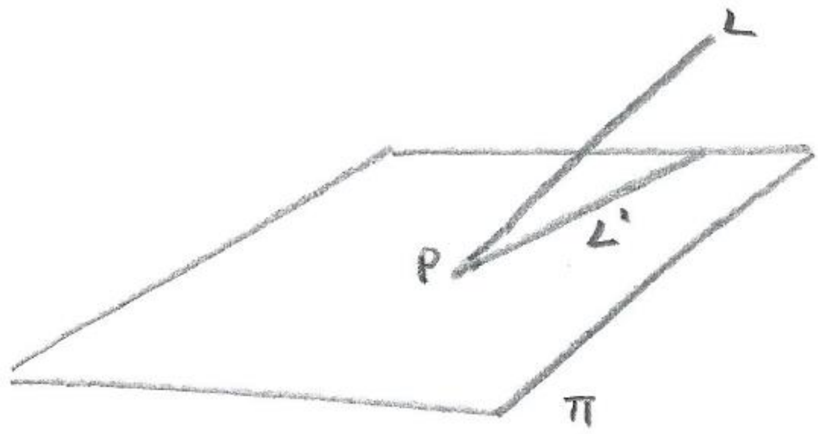
(6) Problem

Use vectors to prove that the mid-points of the sides of any quadrilateral form the vertices of a parallelogram.

(7) Distance from point to plane

Show that the shortest distance from the point $\underline{p}$ to the plane $\underline{r}.\underline{n} = d$ is $\frac{|d-\underline{p}.\underline{n}|}{|\underline{n}|}$

(8) Given the plane Π: $3x + 2y - z = 6$ and the line $L: \underline{r} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$, let L' be the projection of L onto Π



(i) Find the point of intersection (P) of Π & L

(ii) Find the angle between Π & L

(iii) Find a vector that is parallel to Π and perpendicular to L

(iv) Find a vector equation for L'

(v) Find the angle between L and L'

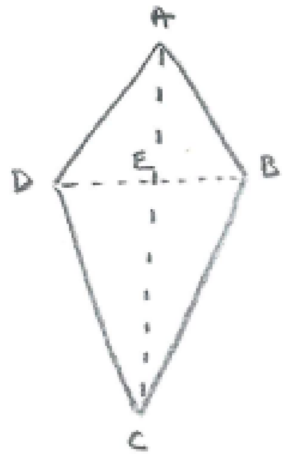
(9)(i) Find the intersection of the line $\underline{r} = \underline{a} + t\underline{b}$ and the plane $\underline{r}.\underline{n} = d$

(ii) Find the intersection of the line $\underline{r} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and the plane $\underline{r}.\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = -2$

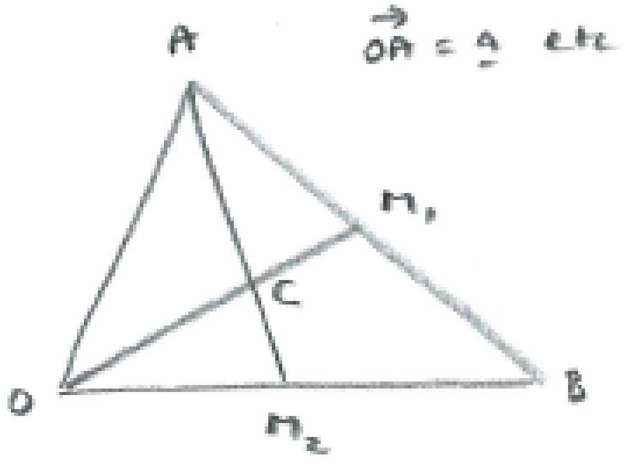
(10) [AEA, June 2009, Q7(d)]

In the diagram below, ABCD is a kite. Find $\overrightarrow{OD}$ if $\overrightarrow{OA} = \begin{pmatrix} -1 \\ 4/3 \\ 7 \end{pmatrix}$,

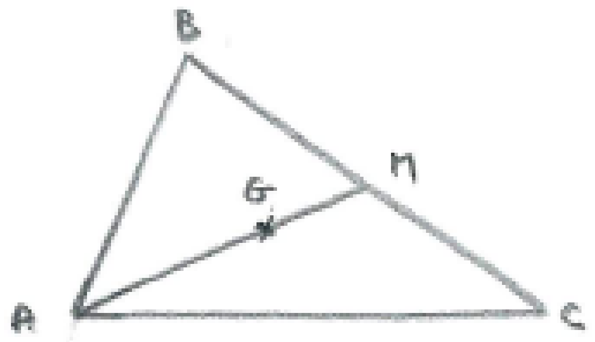
$\overrightarrow{OB} = \begin{pmatrix} 4 \\ 4/3 \\ 2 \end{pmatrix}$ & $\overrightarrow{OC} = \begin{pmatrix} 6 \\ 16/3 \\ 2 \end{pmatrix}$



(11) Prove that the centre of mass of a triangular lamina lies 2/3 of the way along any of the medians.



(12) Given that the centre of mass of a triangular lamina lies 2/3 of the way along any of the medians, prove that it has position vector $\frac{1}{3}(\underline{a} + \underline{b} + \underline{c})$.



(13) Find the angle between adjacent sloping faces of a right square-based pyramid, where the faces are equilateral triangles (as shown in Figure 1).

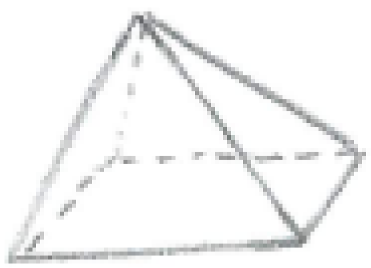


Figure 1