Vectors - Exercises: Part 2 (4 pages; 24/3/20)

Key to difficulty:

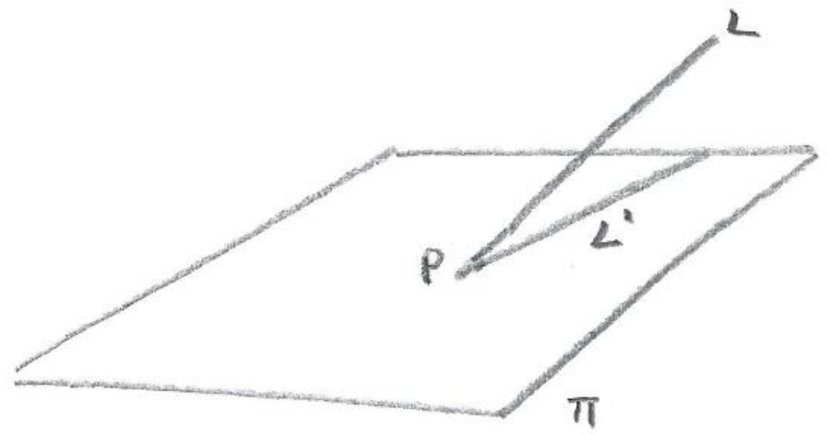
* easier

** moderate

*** harder

(1***) Given the plane Π: $3x + 2y - z = 6$ and the line

$L: \underline{r} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$, let L' be the projection of L onto Π



(i) Find the point of intersection (P) of Π & L

(ii) Find the angle between Π & L

(iii) Find a vector that is parallel to Π and perpendicular to L

(iv) Find a vector equation for L'

(v) Find the angle between L and L'

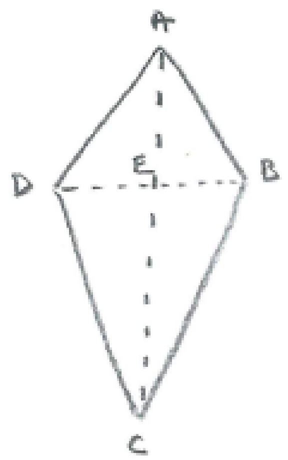
(2***)(i) Find the intersection of the line $\underline{r} = \underline{a} + t\underline{b}$ and the plane $\underline{r}.\underline{n} = d$

(ii) Find the intersection of the line $\underline{r} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and the plane $\underline{r}.\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = -2$

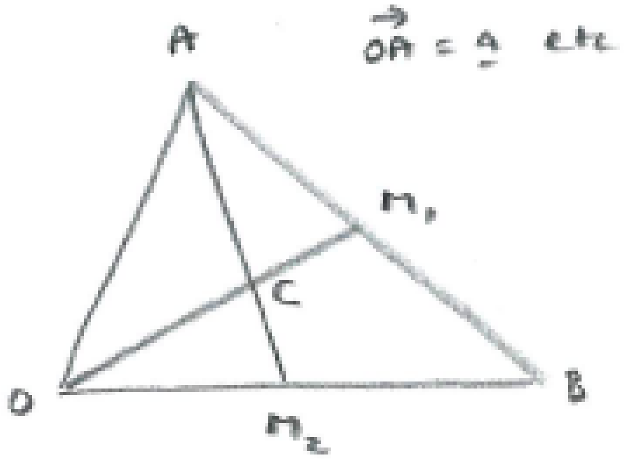
(3***) [AEA, June 2009, Q7(d)]

In the diagram below, ABCD is a kite. Find $\overrightarrow{OD}$ if $\overrightarrow{OA} = \begin{pmatrix} -1 \\ 4/3 \\ 7 \end{pmatrix}$,

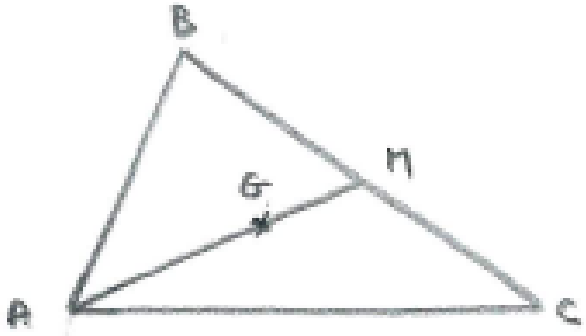
$\overrightarrow{OB} = \begin{pmatrix} 4 \\ 4/3 \\ 2 \end{pmatrix}$ & $\overrightarrow{OC} = \begin{pmatrix} 6 \\ 16/3 \\ 2 \end{pmatrix}$



(4***) Prove that the centre of mass of a triangular lamina lies 2/3 of the way along any of the medians.



(5***) Given that the centre of mass of a triangular lamina lies 2/3 of the way along any of the medians, prove that it has position vector $\frac{1}{3}(\underline{a} + \underline{b} + \underline{c})$.



(6***) Find the angle between adjacent sloping faces of a right square-based pyramid, where the faces are equilateral triangles (as shown in Figure 1).

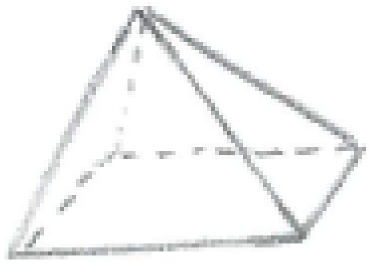


Figure 1