Discrete Uniform Distribution (2 pages; 20/3/20)

(1) $X\sim$ discrete $U(1,n) \Rightarrow$

$P(X=r)=\frac{1}{n}$ for $r=1,2,3,\dots,n$

$=0$ elsewhere

$X\sim$ discrete $U(a,b) \Rightarrow$

$P(X=r)=\frac{1}{b-a+1}$ for $r=a,a+1,a+2,\dots,b$

$=0$ elsewhere

(where a & b are integers)

[If the possible values of X are not consecutive integers, then the notation $U(a,b)$ can't be used.]

(2) $E(X)=\sum_{r=1}^{n} r.\frac{1}{n}=\frac{1}{n}\sum_{r=1}^{n} r=\frac{1}{n}.\frac{1}{2}n(n+1)=\frac{1}{2}(n+1)$

(3) $E(X^2)=\sum_{r=1}^{n} r^2.\frac{1}{n}=\frac{1}{n}.\frac{1}{6}n(n+1)(2n+1)$

$=\frac{1}{6}(n+1)(2n+1)$

Then $Var(X)=E(X^2)-[E(X)]^2$

$=\frac{1}{6}(n+1)(2n+1)-(\frac{n+1}{2})^2$

$=\frac{1}{12}(n+1)\{2(2n+1)-3(n+1)\}$

$=\frac{1}{12}(n+1)(n-1)$ or $\frac{1}{12}(n^2-1)$

(4) $X\sim$ discrete $U(1,n) \Rightarrow$

$P(X \leq r) = \frac{r}{n}$ for $r \in \{1, 2, 3, \dots, n\}$

(5) For eg $Y \in \{5, 7, 9, \dots, 21\}$, we can write $Y = 2X + 3$,

where $X \in \{1, 2, 3, \dots, 9\}$,

so that $E(Y) = 2E(X) + 3$ and $Var(Y) = 4Var(X)$