Uniform (Discrete) Q1[8 marks] (13/6/21)

Exam Boards

OCR : Statistics (Year 1)
MEI: Statistics a
AQA: Statistics (Year 1)
Edx: A Level (Year 1)

The probability distribution of a discrete random variable is shown below.

| $x$ | $a$ | $a+d$ | $a+2 d$ |
| :---: | :---: | :---: | :---: |
| $P(X=x)$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ |

where $a$ and $d$ are positive integers.
Given that $E(X)=6$ and $\operatorname{Var}(X)=6$, find $a$ and $d$.[8 marks]

## Solution

Let $Y$ have the uniform distribution with values $1,2 \& 3$.
[1 mark]
Then $E(Y)=\frac{3+1}{2}=2$ [1 mark]
and $\operatorname{Var}(Y)=\frac{1}{12}\left(3^{2}-1\right)=\frac{2}{3}$ [1 mark]
And $X=d Y+a-d$, [1 mark]
so that $E(X)=d E(Y)+a-d$ [1 mark]
and $\operatorname{Var}(X)=d^{2} \operatorname{Var}(Y)$ [1 mark]
Hence $6=d(2)+a-d ; a+d=6$
and $6=d^{2}\left(\frac{2}{3}\right) ; d=3$ [1 mark], so that $a=3$ [1 mark]

