Trigonometry – Small angle approximations

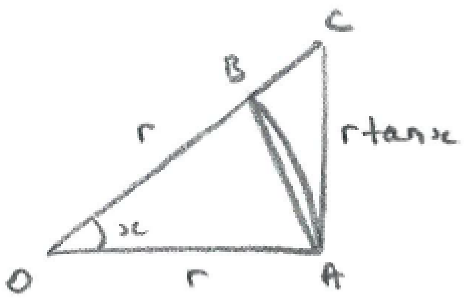
(2 pages; 15/4/21)

These can be derived for $sinx$ & $cosx$ from their Maclaurin expansions [See " Maclaurin Series"].

Thus $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ ...\ \Rightarrow sinx \approx x$ for small x,

& $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ ...\ \Rightarrow \cos x \approx 1 - \frac{x^2}{2}$ for small x

Alternative derivation



For $sinx \approx x$:

$\triangle\, OAB < sectorOAB < \triangle\, OAC$,

so that $\frac{1}{2}r^2 sinx < \frac{1}{2}r^2 x < \frac{1}{2}r(rtanx)$

and hence $1 < \frac{x}{sinx} < \frac{1}{cosx}$

As $x \to 0, \frac{1}{cosx} \to 1$ (from above);

ie we can make $\frac{1}{cosx}$ as close to 1 as we please,

and then $\frac{x}{sinx} \approx 1$; ie $sinx \approx x$ for small x

For $\cos x \approx 1 - \frac{x^2}{2}$:

Starting with $cos(2\theta) = cos^2\theta - sin^2\theta = 1 - 2sin^2\theta$,

and writing $x = 2\theta$, we have $cosx = 1 - 2sin^2\left(\frac{x}{2}\right)$

Then, for small x, $cosx \approx 1 - 2\left(\frac{x}{2}\right)^2 = 1 - \frac{x^2}{2}$