

Trigonometry – Factor Formulae (1 page; 15/4/21)

$$\sin\theta + \sin\phi = 2\sin X \cos Y$$

$$\sin\theta - \sin\phi = 2\cos X \sin Y$$

$$\cos\theta + \cos\phi = 2\cos X \cos Y$$

$$\cos\theta - \cos\phi = -2\sin X \sin Y$$

where $X = \frac{1}{2}(\theta + \phi)$ & $Y = \frac{1}{2}(\theta - \phi)$

Proofs

Let $\theta = X + Y$ & $\phi = X - Y$

Then $\sin\theta + \sin\phi = \sin X \cos Y + \cos X \sin Y + \sin X \cos Y - \cos X \sin Y$
 $= 2\sin X \cos Y$, with X & Y as above; and similarly for the other formulae.