TMUA Methods – Ideas & Exercises (51 Pages; 10/1/25)

MAT 2007, Q1(D)

D. The point on the circle

$$(x-5)^2+(y-4)^2=4$$

which is closest to the circle

$$(x-1)^2+(y-1)^2=1$$

is

(a) $(3.4, 2.8)$, (b) $(3, 4)$, (c) $(5, 2)$, (d) $(3.8, 2.4)$.

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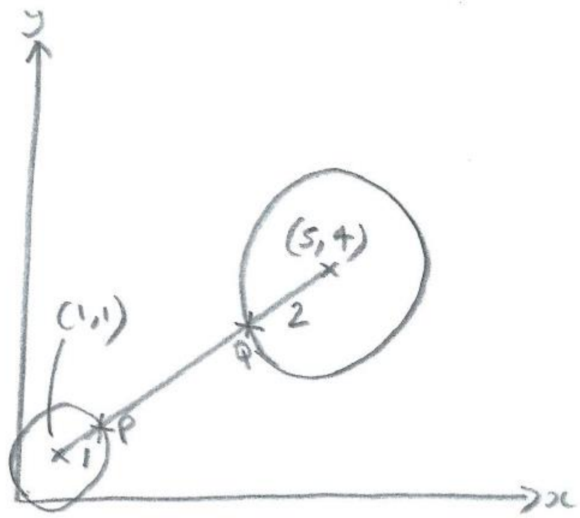
Solution



The distance between the two centres is 5 (by Pythagoras), and the required point is $\frac{2}{5}$ of the way along the line joining the centres, from the point (5,4).

Taking a weighted average of the two centres [' linear interpolation']:

$\frac{2}{5}(1,1)+\frac{3}{5}(5,4)=(\frac{17}{5},\frac{14}{5})$ or $(3.4, 2.8)$

So the answer is (a).

Comments

Simplifying features (3,4,5 triangle) emerges only after diagram has been drawn.

Example of use of linear interpolation.

Sketch the graph of $\sqrt{x^2 - 2x + 1}$ for $0 \leq x \leq 2$

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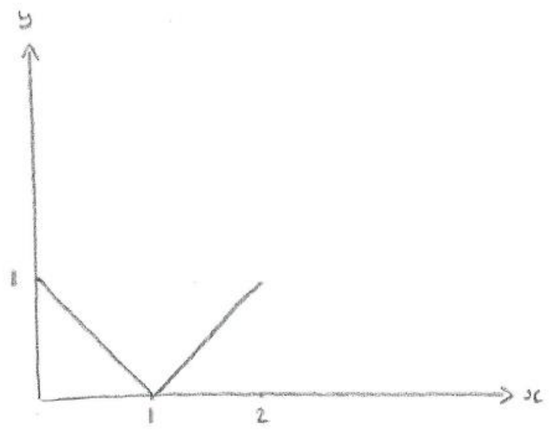
Solution



For $0 \leq x \leq 1$, $\sqrt{x^2 - 2x + 1} = \sqrt{(x-1)^2} = \sqrt{(1-x)^2} = 1 - x$

For $1 \leq x \leq 2$, $\sqrt{x^2 - 2x + 1} = \sqrt{(x-1)^2} = x - 1$

Comments

Example of Case by Case approach.

How many solutions are there to $x^3 - 6x^2 + 9x + 2 = 0$?

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Solution

$\Leftrightarrow x(x^2 - 6x + 9) = -2$

$\Leftrightarrow x(x - 3)^2 = -2$

So one solution, from graph of $y = x(x - 3)^2$

Comments

Example of rearrangement, and reformulation of problem (as graphical rather than algebraic problem).

MAT, Specimen Paper B, Q1/G [Digits]

G. The four digit number 2652 is such that any two consecutive digits from it make a multiple of 13. Another number N has this same property, is 100 digits long, and begins in a 9. What is the last digit of N?

(a) 2 (b) 3 (c) 6 (d) 9

G. The four digit number 2652 is such that any two consecutive digits from it make a multiple of 13. Another number N has this same property, is 100 digits long, and begins in a 9. What is the last digit of N?

(a) 2 (b) 3 (c) 6 (d) 9

Solution

The two digit multiples of 13 are 13, 26, 39, 52, 65, 78 & 91

(which doesn't eliminate any of the suggested answers).

Given that the 1st digit is 9, the 2nd digit must be 1; the 3rd digit 3, and the 4th digit 9, so that we have the cycle 913. This accounts for the first 99 digits, so that the last digit must be 9.

ie the answer is (d)

Comments

Look for something that is quick to do (and likely to be worthwhile); ie consider multiples of 13.

“100 digits long”: suggests looking for a pattern

MAT 2009, Q1/D

D. The smallest positive integer n such that

$$1 - 2 + 3 - 4 + 5 - 6 + \cdots + (-1)^{n+1} n \geqslant 100,$$

is

(a) 99, (b) 101, (c) 199, (d) 300.

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$$1-2+3-4+5-6+\cdots+(-1)^{n+1}n \geqslant 100,$$

is

(a) 99, (b) 101, (c) 199, (d) 300.

Solution

[Because of the presence of $(-1)^{n+1}$, it is worth considering separately even and odd n.]

With even n, the LHS becomes $1-2+3-4+\cdots-2m$, writing $n=2m$.

By grouping the terms as $(1-2)+(3-4)+\cdots-2m$, we see that this has a negative value.

So n must be odd, and the LHS becomes

$1-2+3-4+\cdots+(2m+1)$, writing $n=2m+1$

And the terms can be grouped to give

$(1-2)+(3-4)+\cdots([2m-1]-2m)+(2m+1)$

$=m(-1)+(2m+1)=m+1$

So we want $m+1\geq 100$, and hence

$n=2m+1\geq 2(99)+1=199$

So the answer is (c).

Comments

Example of Case by Case.

Experimenting: eliminates even n

Find all positive integer solutions of the equation

$xy - 8x + 6y = 90$

Find all positive integer solutions of the equation

$xy - 8x + 6y = 90$

Solution

$xy - 8x + 6y = (x+6)(y-8) + 48,$

so that the original equation is equivalent to

$(x+6)(y-8) = 42$

The positive integer solutions are given by:

$x + 6 = 7, y - 8 = 6$

$x + 6 = 14, y - 8 = 3$

$x + 6 = 21\ y - 8 = 2$

$x + 6 = 42, y - 8 = 1,$

so that the solutions are:

$x = 1, y = 14$

$x = 8, y = 11$

$x = 15, y = 10$

$x = 36, y = 9$

Comments

- Consider a simpler problem; eg $xy = 90$

- Standard idea for integer-related questions is factorisation.

- Re-read question: “Find all **positive** integer solutions”.

- Consider ALL cases (eg $x + 6 = -7, y - 8 = -6$)

Can n^3 equal $n + 12345670$ (where n is a positive integer)?

Can n^3 equal $n + 12345670$ (where n is a positive integer)?

Solution

Rearrange to $n^3 - n = 12345670$

$n^3 - n = n(n^2 - 1) = n(n-1)(n+1)$

One of these factors must be a multiple of 3; whereas 12345670 is not a multiple of 3 (since $1 + 2 + 3 + 4 + 5 + 6 + 7 + 0$ isn't a multiple of 3); so answer is No.

Comments

- Large number means that trial and error can probably be ruled out.

- Example of rearrangement.

- Standard idea for integer-related questions is factorisation.

- Useful idea (division by 3) only emerges once you have experimented.

MAT 2007, Q1/J

J. The inequality

$$(n+1)+(n^4+2)+(n^9+3)+(n^{16}+4)+\cdots+(n^{10000}+100)>k$$

is true for all $n \geqslant 1$. It follows that

(a) $k < 1300$,
(b) $k^2 < 101$,
(c) $k \geqslant 101^{10000}$,
(d) $k < 5150$.

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(a) $k < 1300$,
(b) $k^2 < 101$,
(c) $k \geqslant 101^{10000}$,
(d) $k < 5150$.

Solution

Consider $n = 1$ [as this is fairly quick to do]

The LHS is $100 + \frac{1}{2}(100)(101) = 5150$

As the LHS increases with n, the inequality will hold provided that $k < 5150$

So the answer is (d).

Comments

- Experiment with a particular value.

MAT 2008, Q1/B

B. Which is the smallest of these values?

(a) $\log_{10} \pi$, (b) $\sqrt{\log_{10}(\pi^2)}$, (c) $\left(\frac{1}{\log_{10} \pi}\right)^3$, (d) $\frac{1}{\log_{10} \sqrt{\pi}}$.

B. Which is the smallest of these values?

(a) $\log_{10}\pi$, (b) $\sqrt{\log_{10}(\pi^2)}$, (c) $\left(\frac{1}{\log_{10}\pi}\right)^3$, (d) $\frac{1}{\log_{10}\sqrt{\pi}}$.

Solution

Write $L = log_{10}\pi$

[It often helps to have a rough idea of the sizes of the multiple choice options.]

$L \approx \frac{1}{2}$, so that $(b) = \sqrt{2L} \approx 1, (c) = \left(\frac{1}{L}\right)^3 = 8, (d) = \frac{1}{\frac{1}{2}L} \approx 4$

so that we can be fairly sure that the answer is (a), and might just like to check that (a) < (b):

result to prove: $L < \sqrt{2L}$

As $L < 1$ (as $\pi < 10$), $\sqrt{2L} = \sqrt{2}\sqrt{L} > \sqrt{2}\,L > L$, as required.

So the answer is (a).

Comments

Use of approximate values

MAT 2009, Q1/J

J. The number of *pairs* of *positive integers* x, y which solve the equation

$$x^3 + 6x^2y + 12xy^2 + 8y^3 = 2^{30}$$

is

(a) 0, (b) 2^6, (c) $2^9 - 1$, (d) $2^{10} + 2$.

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(a) 0, (b) 2^6, (c) $2^9 - 1$, (d) $2^{10} + 2$.

Solution

The presence of $8y^3$ suggests that $(x + 2y)^3$ might possibly expand to give the LHS - which it does.

This then gives $x + 2y = 2^{10}$, and we can simplify matters by writing $x = 2u$ (since x has to be even), to give $u + y = 2^9$.

Then y can take the values $1, 2, \ldots, 2^9 - 1$ (with $x = 2^{10} - 2y$), so that there are $2^9 - 1$ such pairs.

So the answer is (c).

Comments

Anything complicated-looking is likely to have a simple interpretation.

MAT 2008, Q1/J [Trigonometry]

J. In the range $0 \leqslant x < 2\pi$ the equation

$$(3 + \cos x)^2 = 4 - 2\sin^8 x$$

has

(a) 0 solutions, (b) 1 solution, (c) 2 solutions, (d) 3 solutions.

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Introduction

This equation can be interpreted as the intersection of the functions $y = (3 + cosx)^2$ and $y = 4 - 2sin^8x$.

In order for the functions to intersect, their ranges must overlap.

Given the relatively complicated nature of these functions, the simplest outcome for this question would be that either the ranges don't overlap at all, or they overlap at one value.

Solution

Note that $(3 + cosx)^2 \geq 4$,

whilst $4 - 2sin^8x \leq 4$

So a sol'n only exists when $cosx = -1$ and $sinx = 0$;

ie at $x = \pi$ (in the range $0 \leq x < 2\pi$)

So the answer is (b).

TMUA, Specimen P1, Q7

7. A bag contains n red balls, n yellow balls, and n blue balls.

One ball is selected at random and not replaced.

A second ball is then selected at random and not replaced.

Each ball is equally likely to be chosen.

The probability that the two balls are **not** the same colour is

A $\frac{n-1}{3n-1}$

B $\frac{2n-2}{3n-1}$

C $\frac{2n}{3n-1}$

D $\frac{(n-1)^3}{27(3n-1)^3}$

E $\frac{3(n-1)}{3n-1}$

F $\frac{n^3}{27(3n-1)^3}$

Solution

$P(\text{Balls are not the same colour})$

$= 1 - P(\text{Balls are the same colour})$

$= 1 - 3P(\text{Both balls are Red})$

$= 1 - 3\left(\frac{1}{3}\right)\left(\frac{n-1}{3n-1}\right)$

$= \frac{(3n-1)-(n-1)}{3n-1} = \frac{2n}{3n-1}$

Answer: C

Comments

$P(A') = 1 - P(A)$

Symmetry: $P(\text{Balls are the same colour}) = 3P(\text{Both balls are Red})$

MAT 2009, Q1/C

C. Given a real constant c, the equation

$$x^4 = (x-c)^2$$

has four real solutions (including possible repeated roots) for

(a) $c \leqslant \frac{1}{4}$, (b) $-\frac{1}{4} \leqslant c \leqslant \frac{1}{4}$, (c) $c \leqslant -\frac{1}{4}$, (d) all values of c.

C. Given a real constant c, the equation

$$x^4 = (x-c)^2$$

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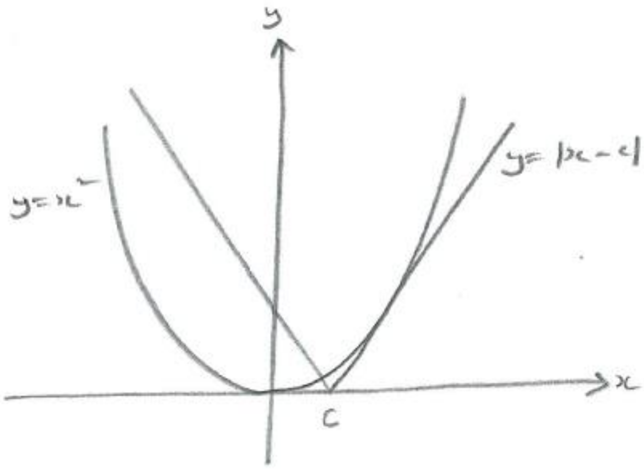
(a) $c \leqslant \frac{1}{4}$, (b) $-\frac{1}{4} \leqslant c \leqslant \frac{1}{4}$, (c) $c \leqslant -\frac{1}{4}$, (d) all values of c.

Solution

$x^4 = (x-c)^2 \Rightarrow x^2 = |x-c|$



The diagram show the critical point at which the number of roots changes from 4 to 2 (for larger values of c). By symmetry, if the critical value of c is c_1, then $-c_1$ will also be a critical value (with the number of roots being 2 for $c < -c_1$. As there is only one answer of the form $-c_1 \leq c \leq c_1$,

the answer must be (b).

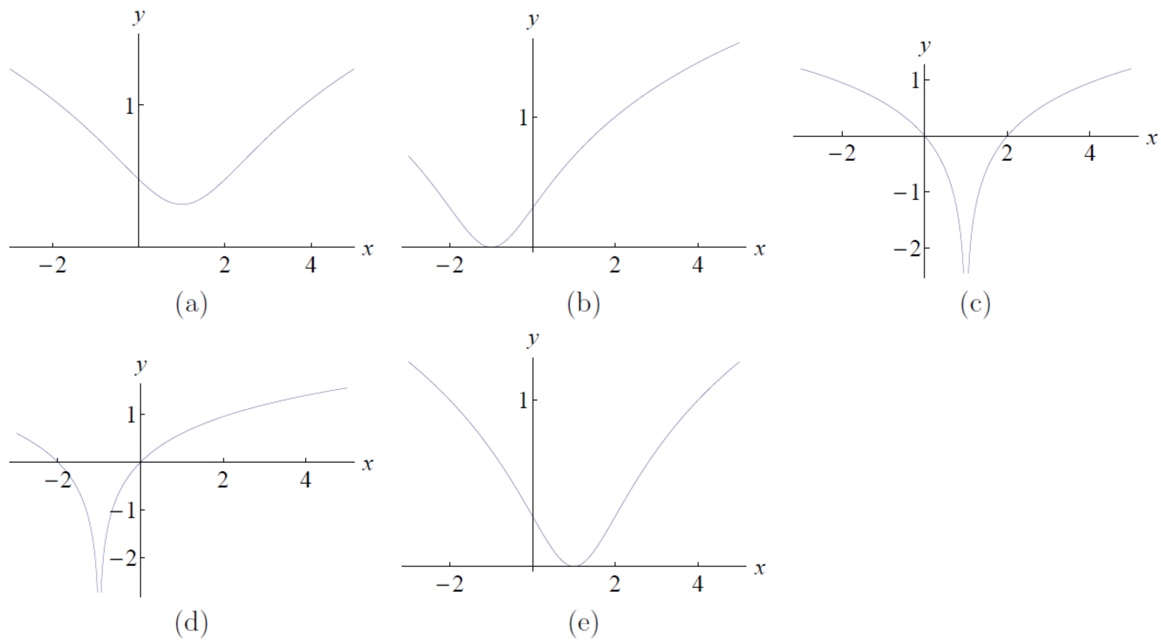
[As a check, the gradients of $y = x^2$ & $y = x - c$ are equal when $2x = 1$; ie $x = \frac{1}{2}$, so that the line $y = x - c$ has to pass through the point $(\frac{1}{2}, \left(\frac{1}{2}\right)^2)$, and hence $\frac{1}{4} = \frac{1}{2} - c$, giving $c = \frac{1}{4}$]

Comment

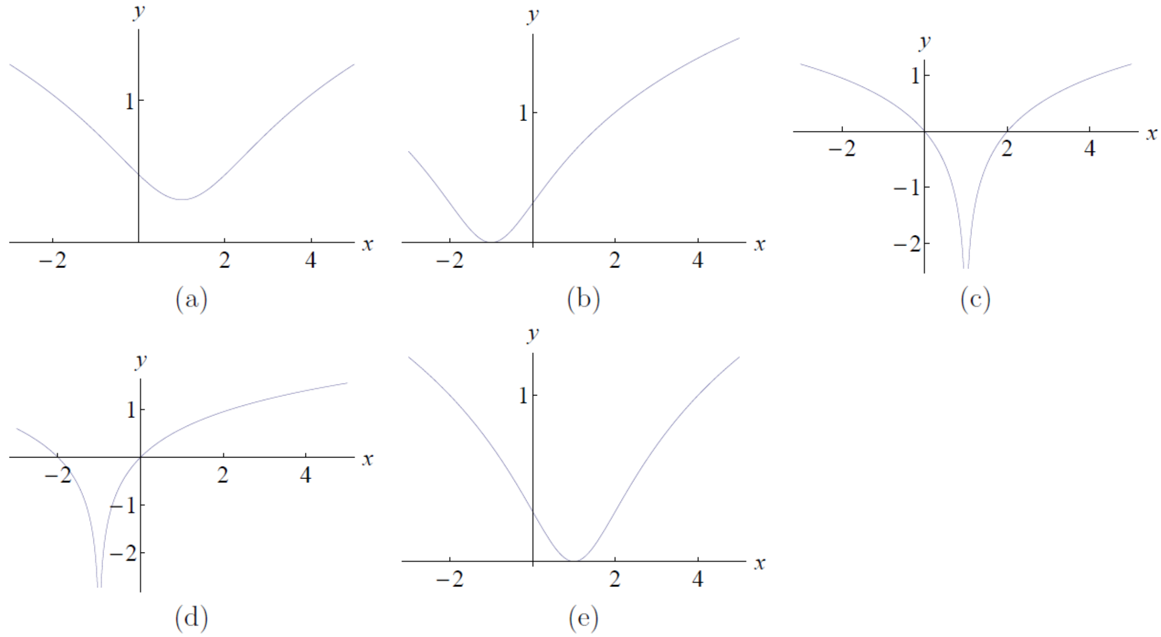
Consider the form of the answer.

MAT 2014, Q1/B

B. The graph of the function $y = \log_{10}(x^2 - 2x + 2)$ is sketched in



B. The graph of the function $y = \log_{10}(x^2 - 2x + 2)$ is sketched in



Solution

[Completing the square is something that is quick to do (and may shed some light on the problem).]

Let $f(x) = log_{10}(x^2 - 2x + 2) = log_{10}[(x-1)^2 + 1]$

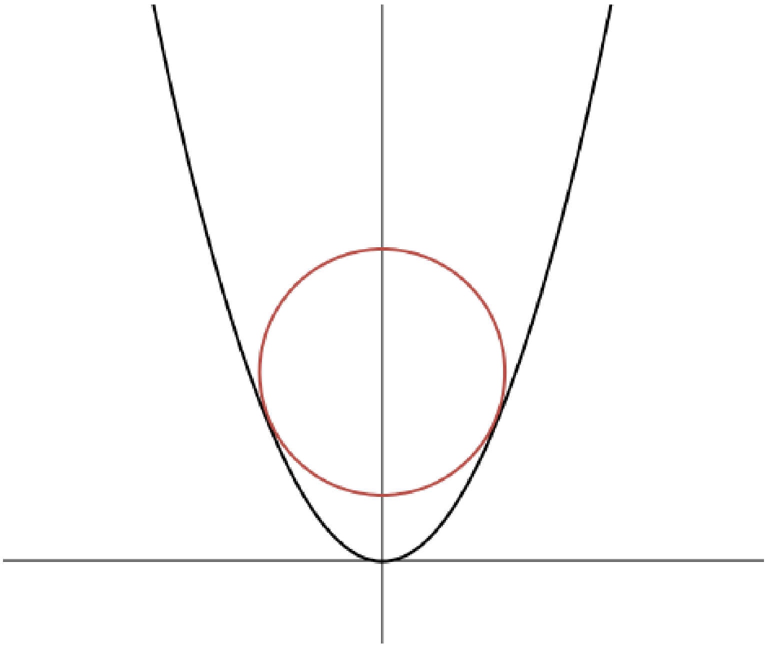
Then $f(1) = 0$, so that (a), (b), (c) & (d) can be eliminated.

Thus the answer is (e).

Comment

Look for things that are quick to do (and likely to be useful).

9 A circle of radius r is *dropped* into the curve $y = x^2$ such that it only touches the curve.



The circle's center is $(0, c)$
Express c in terms of r.

A $\dfrac{1+2r^2}{2}$

B $\dfrac{1-4r^2}{4}$

C $\dfrac{4r}{r^2-2}$

D $\dfrac{r^2}{4}-1$

E $\dfrac{1+4r^2}{4}$

F $\dfrac{1-6r^2}{2}$

G $\dfrac{r^3-r}{4}$

Solution

Consider the points of intersection of the circle

$x^2 + (y - c)^2 = r^2$ with $y = x^2$,

given by $y + (y - c)^2 = r^2$,

or $y^2 + (1 - 2c)y + c^2 - r^2 = 0$ (*)

In order for the circle to be touching the curve (as in the diagram), there must be a single value of y satisfying (*) [producing the points $(-\sqrt{y}, y)$ & $(\sqrt{y}, y)$];

ie the discriminant of (*) must be zero:

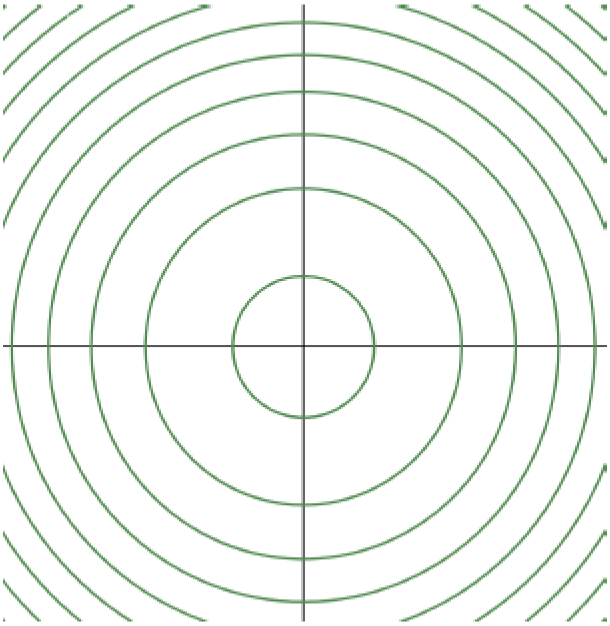
$(1 - 2c)^2 - 4(c^2 - r^2) = 0$;

$1 - 4c + 4r^2 = 0$;

$c = \frac{1+4r^2}{4}$

Answer: E

14 The graph of $\tan(x^2 + y^2) = 1$ is shown below (angles in radians).



The graph consists of a central circle surrounded by infinitely many **annuluses** or rings.

The central circle has area C and each annulus has the same constant area A.

What is $\frac{A}{C}$?

[An *annulus* is the region between two concentric circles (like a ring)]

A 4

B $\frac{1}{2}$

C π^2

D $\frac{3}{4\pi}$

E $\frac{\pi^2+1}{4\pi}$

F $\frac{1}{\pi}$

Solution

The successive circles have equations $x^2 + y^2 = \frac{\pi}{4} + n\pi$, where $n = 0, 1, 2, \ldots$

Then $C = \pi(\frac{\pi}{4})$ and $A = \pi\left(\frac{\pi}{4} + n\pi\right) - \pi\left(\frac{\pi}{4} + [n-1]\pi\right) = \pi^2$;

so that $\frac{A}{C} = 4$

Answer: A

16 a, b and c are positive integers so that

$$\frac{\left(\frac{a}{c}+\frac{a}{b}+1\right)}{\left(\frac{b}{a}+\frac{b}{c}+1\right)}=17$$

How many solutions are there to the inequality

$$a+2b+3c\leq 50$$

A 14

B 15

C 16

D 17

E 18

F 19

G 0

Solution

$$\frac{\frac{a}{c}+\frac{a}{b}+1}{\frac{b}{a}+\frac{b}{c}+1}=17 \Leftrightarrow a^2b+a^2c+abc=17(b^2c+b^2a+abc)$$

$\Leftrightarrow a(ab+ac+bc)=17b(bc+ba+ac)$

$\Leftrightarrow a=17b$ (as $ab+ac+bc\neq 0$)

Case 1: $\boldsymbol{b=1}$

$\Rightarrow a=17$; then $a+2b+3c\leq 50\Rightarrow 3c\leq 31\Rightarrow\ c=1,2,\dots,10$

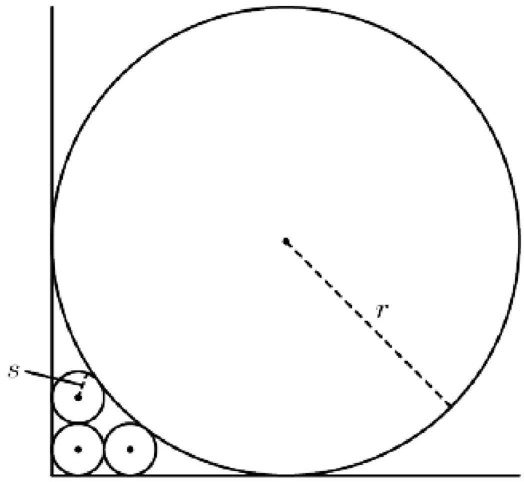
Case 2: $\boldsymbol{b=2}$

$\Rightarrow a=34$; then $a+2b+3c\leq 50\Rightarrow 3c\leq 12\Rightarrow\ c=1,2,3,4$

So there are $10+4=14$ possibilities

Answer: A

17 Three circles of radius s are drawn in the first quadrant of the xy-plane. The first circle is tangent to both axes, the second is tangent to the first circle and the x-axis, and the third is tangent to the first circle and the y-axis. A circle of radius $r > s$ is tangent to both axes and to the second and third circles. What is r/s?



A 5

B 6

C 8

D 9

E 10

F 12

Solution

Let the centre of the circle nearest the Origin be D. Let the circle above this one have centre A, and the one to the right of it have centre B. Let the large circle have centre C.

Let E be the midpoint of AB.

Consider the right-angled triangle AEC.

$AC = s + r$

$AE = \frac{1}{2}AB = \frac{1}{2}\left(\sqrt{2}DB\right) = \frac{1}{2}\sqrt{2}(2s) = \sqrt{2}\,s$

$EC = DC - DE = \sqrt{(r-s)^2 + (r-s)^2} - AE$

$= \sqrt{2}(r-s) - \sqrt{2}\,s = \sqrt{2}(r-2s)$

Then, by Pythagoras, $AC^2 = AE^2 + EC^2$,

so that $(s+r)^2 = 2s^2 + 2(r-2s)^2$,

Writing $k = \frac{r}{s}$, $(1+k)^2 = 2 + 2(k-2)^2$,

so that $0 = k^2 - 10k + 9$;

$(k-1)(k-9) = 0$

Then, as $r > s, \frac{r}{s} = 9$

Answer: D

3 What are the solution(s) to

$$\ln(x^2 + 4x - 90) = \ln(-2x + 1)$$

For all real x?

A $13, 7$

B $-13, 7$

C $-13, -7$

D -13

E 7

Solution

Conditions to be satisfied:

$x^2 + 4x - 90 = -2x + 1$ and $-2x + 1 > 0$

ie $x^2 + 6x - 91 = 0$ and $x < \frac{1}{2}$

$x^2 + 6x - 91 = (x + 13)(x - 7)$,

so only $x = -13$ satisfies the required conditions

Answer: D

5 After Euclid High School's last basketball game, it was determined that $\frac{1}{4}$ of the team's points were scored by Alexa and $\frac{2}{7}$ were scored by Brittany. Chelsea scored 15 points. None of the other 7 team members scored more than 2 points. What was the total number of points scored by the other 7 team members?

A 10

B 11

C 12

D 13

E 14

Solution

We need to find X, where $\frac{T}{4}+\frac{2T}{7}+15+X=T$

(T being the total of the team's points),

given that each of D to J is limited to 2, so that $X \leq 14$

Because each person's points has to be an integer, we can write

$T = 28M$, where M is a positive integer.

Then $7M+8M+15+X=28M$,

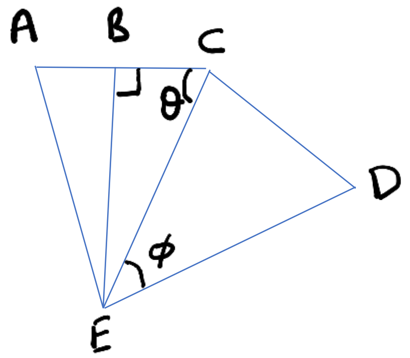
so that $15+X=13M$ (with $X \leq 14$)

This forces X to be 11.

Answer: B

Let $A(n)$ be the area of a regular polygon with n sides, each of length 1, where $n \geq 3$. Show that $A(n) = \frac{n}{4} cot(\frac{180}{n}°)$

Solution



Referring to the diagram (where AC & BD are adjacent sides of the polygon), $\phi = \frac{360}{n}$; $\theta = 90 - \frac{\phi}{2} = 90 - \frac{180}{n}$

Area of BCE is $\frac{1}{2}BC.BE = \frac{1}{2}\left(\frac{1}{2}\right)BCtan\theta$

$= \frac{1}{4}\left(\frac{1}{2}\right)\tan\,(90 - \frac{180}{n})°$

$= \frac{1}{8}\cot\,(\frac{180}{n})°$

Then $A(n) = n \times 2 \times$ Area of BCE $= \frac{n}{4}\cot\,(\frac{180}{n})$

8 Which of the following expressions is closest to the value of

$$\sqrt{100001} - \sqrt{100000}$$

A $\dfrac{1}{\sqrt{100000}}$

B $\dfrac{1}{2\sqrt{100000}}$

C $\dfrac{1}{10\sqrt{100000}}$

D $\dfrac{1}{100\sqrt{100000}}$

E $\dfrac{1}{100000\sqrt{100000}}$

Solution

Consider $(\sqrt{100001} - \sqrt{100000})(\sqrt{100001} + \sqrt{100000})$

$= 100001 - 100000 = 1$

Then $\sqrt{100001} - \sqrt{100000} = \frac{1}{\sqrt{100001}+\sqrt{100000}}$

$\approx \frac{1}{2\sqrt{100000}}$

[This is clearly closer than any of the other options.]

Answer: B

10 Simplify

$$\sqrt{3 - 2\sqrt{2}}$$

A $\sqrt{2}$

B $2 + \sqrt{2}$

C $\sqrt{2} - 1$

D $2 - \sqrt{2}$

E $1 - \sqrt{2}$

F $\dfrac{\sqrt{2}}{2} + \dfrac{1}{2}$

G $3 - 3\sqrt{2}$

Solution

[Assuming that this problem is not intended to be too hard,]

Can we write $3 - 2\sqrt{2}$ as a perfect square?

Perhaps of the form $(a + b\sqrt{2})^2$?

This would require $2ab = -2$ and $3 = a^2 + 2b^2$

These equations could be solved, but it's worth looking for simple solutions first. Thus $a = 1, b = -1$ and $a = -1, b = 1$ work.

So we have a choice between $\sqrt{(1 - \sqrt{2})^2}$ and $\sqrt{(-1 + \sqrt{2})^2}$

But we require a positive number for the square root, and so

$\sqrt{3 - 2\sqrt{2}} = -1 + \sqrt{2}$ or $\sqrt{2} - 1$

[Alternatively, we could deduce from the forms of the options given, that $\sqrt{3 - 2\sqrt{2}} = a + b\sqrt{2}$]

Answer: C

16 a, b and c are positive integers so that

$$\frac{\left(\frac{a}{c}+\frac{a}{b}+1\right)}{\left(\frac{b}{a}+\frac{b}{c}+1\right)}=17$$

How many solutions are there to the inequality

$$a+2b+3c\leq 50$$

A 14

B 15

C 16

D 17

E 18

F 19

G 0

Solution

$$\frac{\frac{a}{c}+\frac{a}{b}+1}{\frac{b}{a}+\frac{b}{c}+1} = 17 \Rightarrow$$

$(a^2b + a^2c + abc) = 17(b^2c + b^2a + abc)$

$\Rightarrow a(ab + ac + bc) = 17b(bc + ba + ac)$

$\Rightarrow a = 17b$ (as $ab + ac + bc \neq 0$)

Then, as $a + 2b + 3c \leq 50$,

With a, b & c being positive integers, the possibilities are:

$b = 1; a = 17;\ c = 1,2, \dots, 10$

$b = 2; a = 34;\ c = 1,2,3,4$

ie there are 14 solutions

Answer: A

19 Find the number of integer values of k in the range $-500 \leq k \leq 500$ for which the equation $\log(kx) = 2\log(x+2)$ has exactly one real solution.

A 499

B 500

C 501

D 502

Solution

We require $kx = (x+2)^2$, as well as $kx > 0$ and $x + 2 > 0$

Thus $k = 0$ isn't possible.

Case 1: $\boldsymbol{k > 0}$

Then, as $kx > 0$ and $x + 2 > 0$, it follows that $x > 0$

Consider intersections of $y = (x+2)^2$ and $y = kx$

There will be 2 intersections for each value of k, except where

$y = kx$ touches $y = (x+2)^2$;

ie when $x^2 + (4-k)x + 4 = 0$ has a repeated root;

ie when $(4-k)^2 - 4(4) = 0$

or $4 - k = \pm 4$,

so that $k = 8$ (as $k > 0$)

Thus, there is just one value of $k > 0$ such that the given equation has exactly one real solution.

Case 2: $\boldsymbol{k < 0}$

Then, as $kx > 0$ and $x + 2 > 0$, it follows that $-2 < x < 0$.

Drawing the graphs shows that, for each value of k, there will be just one point of intersection that satisfies $-2 < x < 0$.

And so there will be 500 integer values of k in the range $[-500, -1]$

Hence the total number of integer values of k for cases 1 & 2 is 501.

Answer: C