TMUA - Exam Technique (10 pages; 9/1/25)

Contents

(A) Introduction

Although there is a specific syllabus for the TMUA paper, you may be able to gain an advantage from additional knowledge. See “TMUA – Extra Material”.

Don't do anything that is too obscure: the correct approach, once found, is usually relatively 'simple'. Always consider the simplest possible interpretation of anything that is unclear about the question.

If a topic looks unfamiliar, remember that knowledge outside the syllabus is not assumed, so the question should be self-contained and include definitions of new concepts. Usually such questions turn out to be easier than normal, as the candidate is being rewarded for coping with an unfamiliar topic.

Whatever exam techniques you adopt, it is probably best to allow them to develop gradually over the course of the preparation period, so that by the time you come to the exam the techniques will have been tested.

(B) Multiple Choice techniques

It may be possible to eliminate some of the Multiple Choice options. However, there is obviously a danger that time may be spent eliminating some of the answers, only to find that a direct approach is needed in order to decide between the remaining options.

Sometimes the correct answer can be deduced from its form only (eg if it has to be of the form $a \leq k \leq b$).

As a general principle, look for (useful) things that are quick to do (ie where you can quickly establish whether they are leading anywhere).

Often a decision will need to be made as to whether an algebraic or graphical approach is appropriate. Bear in mind that a simplifying feature of a problem might only be revealed once a diagram has been drawn.

(C) Approaches & Ideas

(C1) Observations

(1) Identification of simplifying features

(a) When solving $f(x) = g(x)$, it may be the case that $f(x)$ attains a maximum when $g(x)$ attains a minimum.

(b) If n is an integer, it may be the case that a solution only exists for even n , for example.

(c) Use of symmetry

(2) Do the Multiple Choice options give any clues? It may help you to get on the wavelength of the question, or provide inspiration.

(3) Ensure that all of the information provided in the question has been used.

(4) If a unique solution is required, or if there is to be exactly 2 solutions, or no solutions, then this may suggest the solving of a quadratic equation and considering the discriminant $b^2 - 4ac$.

(For example, if a straight line is required to touch a quadratic curve.)

A condition in the form of an inequality can also suggest the use of $b^2 - 4ac$.

(5) If you are told that $x \neq a$, then the solution may involve a division by $x - a$.

(6) The presence of a $\pm$ sign often suggests that a square root is being taken at some stage (eg to solve a quadratic equation).

(C2) Creating equations

(1) Equations can be created from:

(a) information in the question

(b) relevant definitions and theorems

(Look out for standard prompts to use a particular result. For example, a reference to a tangent to a circle can suggest the result that the radius and the tangent are perpendicular.)

If necessary, create your own variables (for example, a particular length in a diagram).

Sometimes the advantage of creating an equation is that it gives you something to manipulate; ie in order to make further progress.

(2) When setting up equations or inequalities:

(i) Use k^2 to represent a positive number

(ii) Use $2k$ to represent an even numbe, and $2k + 1$ to represent an odd number.

(C3) Case by case approach

(1) Example: Solve $\frac{x^2+1}{x^2-1} < 1$

Case 1: $x^2 - 1 < 0$; Case 2: $x^2 - 1 > 0$

(Once we know whether $x^2 - 1$ is positive or negative, we can multiply both sides of the inequality by it, changing the direction of the inequality as necessary.)

(2) Transitional (or 'critical') points

This involves considering the point(s) at which the nature of a problem changes.

Example 1: $\frac{(x-1)(x+2)(x-3)}{(x+1)(x-2)(x+3)} < 0$

The only points at which the sign of the left-hand side can change are at the roots of $(x-1)(x+2)(x-3) = 0$, and at the vertical asymptotes $x = -1, x = 2$ and $x = -3$

(C4) Rearrangement or reformulating of a problem

(1) An equation or expression may be capable of being rearranged into a simpler or more convenient form.

(2) As an example of a reformulation: in order to solve the equation $f(x) = k$, consider where the graph of $y = f(x)$ crosses the line $y = k$.

Or turn an equation into the intersection of a curve and a straight line.

Or, more generally, rearrange an equation to $f(x) = g(x)$, and find where the curves $y = f(x)$ & $y = g(x)$ meet (or show that they won't meet).

(3) To sketch the cubic $y = x^3 + 2x^2 + x + 3$, rewrite it as

$y = x(x^2 + 2x + 1) + 3$, and translate the graph of

$y = x(x^2 + 2x + 1) = x(x + 1)^2$

(4) Pairs of numbers x, y might be represented by the coordinates (x, y); eg to find possible integer values of x & y, determine the grid points within the relevant area.

(5) To show that a function $f(n)$ of an integer n cannot be a perfect square, perhaps show instead that $f(n) - 1$ is always a perfect square.

(6) To show that two functions $f(n)$ & $g(n)$ cannot be equal, perhaps show that they belong to different classes; eg even and odd numbers.

(7) Make a substitution. As a simple example, writing $y = 2^x$ might turn an equation into a quadratic.

(C5) Experimenting

(1) Often inspiration for a particular problem will only come after experimenting (including drawing a diagram); or an important feature of a problem will not become apparent until then.

For example, a sequence may turn out to be periodic.

(2) A complicated expression may often be designed by the question setter to be capable of simplification; eg by factorisation or being a perfect square.

An algebraic equation may be simplified by a fortuitous cancellation.

(3) Drawing a diagram may reveal a hidden feature of a problem; eg a triangle may turn out to be right-angled, or the solution may lie on the boundary of a region.

(4) Try out particular values (eg $n = 1$)

This may reveal a simplifying feature of the problem (eg if an integer n is involved, then perhaps it has to be even).

(5) Consider what happens when $x \to \infty$.

(6) Use approximate values to get a feel for a problem (eg $log_{10}3 \approx \frac{1}{2}$); especially for comparing Multiple Choice answers.

(7) Consider a simpler version of the problem (eg experiment with a simple function such as $y = x^2$).

If investigating a situation involving the intersection of a curve and a straight line, perhaps consider first the case where the line touches the curve (ie is a tangent).

(8) Start to list the terms of a sequence - a pattern may emerge.

For counting problems, find a systematic way of listing the possibilities, and then of counting the items in the list.

(D) Integer problems

(1) It is often worth considering even and odd n separately. For example, a solution may only exist for even n.

(2) If an integer-valued variable is known to be (say) a quarter of another variable, so that $m = \frac{n}{4}$, then n has to be a multiple of 4.

(3) The concept of factorisation may be relevant to integer problems.

Note though that whilst $n^2 + 2n + 2 = (n + 1)^2 + 1$ cannot be factorised for all n, when $n = 2$ we have $n^2 + 2n + 2 = 2 \times 5$.

[See "Reformulating a problem" for further ideas.]

(E) Common Pitfalls

(1) Not considering all cases, or not giving special treatment to certain cases.

(2) Losing a solution of an equation by dividing out a factor.

(3) Multiplying an inequality by a quantity without realising that it is (or could be) negative (eg $\ln\left(\frac{1}{2}\right)$).

(4) A square root has to be non-negative.

Thus $\sqrt{(x-1)^2} = x - 1$ is only possible if $x \geq 1$

(but we could write $\sqrt{(x-1)^2} = |x - 1|$ instead).

(5) Overlooking implied constraints. For example, an equation containing $\log(2x-1)$ will only be valid when $x > \frac{1}{2}$.

(F) Checking

(1) Read over each line of your working before moving on to the next one. This is the most efficient way of picking up any errors.

(2) Just before embarking on a solution, re-read the question. Also re-read it when you think you have finished answering the question, in case there is an additional task that you have forgotten about. It is also a good idea to re-read the question if you find yourself getting bogged down in awkward algebra, or if you don't seem to be getting anywhere.

(G) Use of time

(1) Before embarking on a solution, consider how likely it is that it will work, and how much time it will take.

(2) Save time by using letters to represent recurring expressions (eg "write $y = sin^2x$").

(3) You might like to save a relatively straightforward task to complete in the last few minutes of the exam, rather than frantically looking through the paper for something to check.