Suvat Equations (5 pages; 24/1/17)

(1) Deriving the suvat equations (using Calculus for (i) & (ii))

(i) If the acceleration is constant, then $\frac{dv}{dt} = a$

Integrating wrt t gives $v = at + C$

If $v = u$ when $t = 0$, then $C = u$

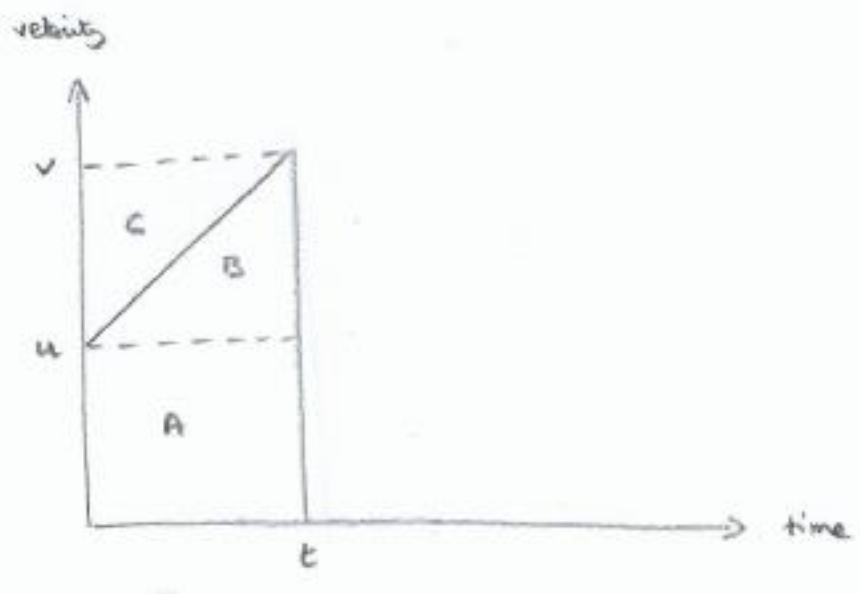
and $v = u + at$ (1)

(ii) Integrating (1) wrt t gives $s = ut + \frac{1}{2}at^2 + C'$

If $s = 0$ when $t = 0$, then $C' = 0$

and $s = ut + \frac{1}{2}at^2$ (2)

(iii) Considering (1) as the equation of a straight line, with v-intercept u and gradient a (see diagram below):



$s =$ area under the graph $= A + B = ut + \frac{1}{2}t(v - u)$

$= ut + \frac{1}{2}t(at)$, from (1) (or the fact that a is the gradient)

So $s = ut + \frac{1}{2}at^2$, agreeing with (2)

(iv) Alternatively, $s = (A + B + C) - C = vt - \frac{1}{2}at^2$ (since $C = B$)

So $s = vt - \frac{1}{2}at^2$ (3)

(v) Also, from the formula for the area of a trapezium (the average of the two heights $\times$ the base):

$s = 1/2(u + v)t$ (4)

(vi) $v = u + at$ and $s = 1/2(u + v)t$

$\Rightarrow v - u = at$ and $u + v = \frac{2s}{t}$

Hence, $(v - u)(v + u) = 2as$

and $v^2 - u^2 = 2as$,

so that $v^2 = u^2 + 2as$ (5)

(2) Choosing the appropriate suvat equation to use

(i) Note that each of the 5 equations is missing one of the suvat variables. The appropriate suvat equation can be established by noting which of the 5 possible variables is not needed (ie the value of the variable is not given, and we are not required to find it).

(ii) In order to find the value of one of the suvat variables, we therefore need 3 pieces of information.

Once the value of the required variable has been found using a particular suvat equation, we now have the values of 4 of the suvat variables.

In order to find the value of the 5th variable, the best method is usually to use the suvat equation that does not involve the first variable that we calculated, in case we made a mistake.

Any of the remaining 3 suvat equations can then be used as a check.

(3) Example: Ball thrown vertically upwards

$u = 10ms^{-1}$

$a = -9.8\,ms^{-2}$

(i) Maximum height $\Rightarrow v = 0$

$v = u + at \Rightarrow 0 = 10 - 9.8t \Rightarrow t = 1.02041 = 1.02\ sec$ (3sf)

[seconds are normally denoted by s, but there is obviously scope for confusion with the displacement]

$v^2 = u^2 + 2as \Rightarrow 0 = 100 - 2(9.8)s$

$\Rightarrow s = \frac{100}{2(9.8)} = 5.10204 = 5.10m$ (3sf)

Alternatively, $s = \frac{1}{2}(10 + 0)(1.02041) = 5.10205$

(ii) Total time in flight (T)

(ie from the moment the ball is projected upwards until the moment it hits the ground again)

By symmetry, $T = 2 \times 1.02041 = 2.04082 = 2.04$ sec (3sf)

(iii) Velocity on hitting the ground (v_T)

By symmetry, $v_T = -10ms^{-1}$

(iv) A possibly useful exercise is to apply each of the 5 suvat equations to the whole of the flight.

[Note that, although the ball changes direction, the acceleration is constant throughout, and so the suvat equations are applicable for the whole flight.]

We see that some of the equations allow us to determine T *or* v_T straightaway:

(A) $\boldsymbol{v = u + at}$

$v_T = u - gT = 10 - 9.8T \quad (1)$

(B) $\boldsymbol{v^2 = u^2 + 2as}$

$(v_T)^2 = (10)^2 - 2(9.8)(0) \Rightarrow v_T = -10 \quad (2)$

(C) $\boldsymbol{s = \frac{1}{2}(u + v)t}$

$s_y = \frac{1}{2}(u + v_T)T \Rightarrow 0 = \frac{1}{2}(10 + v_T)T$

$\Rightarrow v_T = -10$ (3)

(D) $\boldsymbol{s = ut + \frac{1}{2}at^2}$

$s_y = (10)T - \frac{1}{2}gT^2 \Rightarrow 0 = T(10 - 4.9T)$

$\Rightarrow T = \frac{10}{4.9} = 2.04082$ (4)

(E) $\boldsymbol{s = vt - \frac{1}{2}at^2}$

$s_y = v_T T + \frac{1}{2}gT^2 \Rightarrow 0 = T(v_T + 4.9T)$

$\Rightarrow v_T = -4.9T$ (5)