Surds – Exercises (with Sol'ns) (6 pages; 1/2/26)

Simplify $\left(1+\left(1+2^{-\frac{1}{2}}\right)^{-1}\right)^{-1}$

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Solution

$$\left(1+\left(1+2^{-\frac{1}{2}}\right)^{-1}\right)^{-1}=\frac{1}{1+\frac{1}{1+\frac{1}{\sqrt{2}}}}=\frac{1}{1+\frac{\sqrt{2}}{\sqrt{2}+1}}=\frac{\sqrt{2}+1}{\sqrt{2}+1+\sqrt{2}}=\frac{\sqrt{2}+1}{2\sqrt{2}+1}=$$

$$\frac{(\sqrt{2}+1)(2\sqrt{2}-1)}{(2\sqrt{2}+1)(2\sqrt{2}-1)}$$

$$=\frac{4-\sqrt{2}+2\sqrt{2}-1}{8-1}=\frac{3+\sqrt{2}}{7}$$

Find the square roots of $49 - 12\sqrt{5}$

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Solution

Let $x^2 = 49 - 12\sqrt{5}$

Consider $x = a + b\sqrt{5}$

Then $a^2 + 2ab\sqrt{5} + 5b^2 = 49 - 12\sqrt{5}$

Let $a^2 + 5b^2 = 49 \ \& \ 2ab = -12$

[a variation on Equating Coefficients]

Looking for integer solutions, we see that either

$a = 2$ & $b = -3$ or $a = -2$ & $b = 3$ work.

(i) Does $\sqrt{4}$ equal 2 or ± 2? (ii) Simplify $\sqrt{x^2}$

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Solution

(i) By convention, 2 (consider the $\pm$ in the quadratic formula).

(ii) $|x|$