Statistics - Exercises (3 pages; 7/4/20)

Key to difficulty:

* easier

** moderate

*** harder

Binomial distribution

(1**) If $X \sim B(n, p)$, prove that:

(i) $E(X) = np$ (ii) $Var(X) = np(1-p)$

Normal distribution

(2***) Suppose that the heights (in cm) of adult males in the UK are distributed $N(174,49)$.

(i) Assuming that there are 2.5 cm to an inch, what proportion of adult males in the UK are over 6 ft? Give your answer to 1dp.

(ii) In another country, the heights of adult males are distributed Normally, such that 10% are over 6 ft and 5% are under 5ft. Find the mean and variance of the distribution. Give your answers to 1dp.

(3***) Show that 1 standard deviation to either side of the mean of the Normal distribution occurs at the point of inflexion of the Normal curve.

Rank Correlation

(4***) Show that the formula $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$ can be written as $r_s = 1 - \frac{6\sum {d_i}^2}{n(n^2-1)}$

when the data items are ranks.

[In other words, instead of using the formula for Spearman's coefficient, it is theoretically possible to use the standard formula for r.]

Venn Diagrams

(5**) The events A and B are independent and are such that $P(A) = \frac{1}{5}$ and $P(A \cup B) = \frac{1}{2}$

Find:

(i) $P(B)$

(ii) $P(A' \cup B')$

(iii) $P(B' \cap A)$

(iv) $P(B'|A)$

Probability generating functions

(6***) Given that $X_1, X_2, \dots, X_N$ & N are independent random variables, where the X_i are all distributed as X, and that

$S_N = X_1 + X_2 + \cdots + X_N$,

prove that $Var(S_N) = E(N)Var(X) + Var(N)[E(X)]^2$

The following results may be used:

(A) $E(X) = G_X'(1)$

(B) $VarX = G_X''(1) + G_X'(1) - [G_X'(1)]^2$

(C) $G_{S_N}(s) = G_N\big(G_X(s)\big)$

(D) $E(S_N) = E(N)E(X)$

(7***) ['Poisson hen'] A hen lays N eggs, where $N \sim P_o(\lambda)$, and each egg has probability p of hatching. Using any results about probability generating functions , show that the total number of eggs that hatch $\sim P_o(\lambda p)$.