Simplex Algorithm - Part 2 (16 pages; 3/11/16)

(1) The basic Simplex method (covered in Part 1) assumes that all the constraints are of the \leq type (apart from $x \geq 0$, $y \geq 0$).

A constraint such as $3x + 2y - z \ge -2$ (as in Example 3 in Part 1) can just be rewritten as $-3x - 2y + z \le 2$, but the constraint $3x + 2y - z \ge 2$ could not be dealt with in this way, as it would leave us with a negative value on the RHS.

There are two methods for dealing with such a problematic constraint: the 2-Stage Simplex method and the Big M (Simplex) method.

(2) 2-Stage Simplex Method

Example



3

Create slack variables as usual, but introduce an artificial variable for the $x + y \ge 4$ constraint.

$$P - x - y = 0$$
 (1)
 $2x + 3y + s_1 = 12$ (2)

 $6x + 5y + s_2 = 30 (3)$ $x + y - s_3 + a_1 = 4 (4) (s_1, s_2, s_3, a_1 \ge 0)$

 a_1 is needed because $x + y - s_3 = 4$ doesn't allow x = y = 0, since $s_3 \ge 0$; but with the artificial variable we can now start with $x = y = s_3 = 0$ & $a_1 = 4$

Initial solution:

 $x = y = s_3 = 0$; $s_1 = 12$, $s_2 = 30$, $a_1 = 4$, P = 0

The aim is to minimise a_1 , so that (if possible) the solution moves into the feasible region.

Create a new objective: minimise $A = a_1$

[It might seem a bit unnecessary to create another variable with the same value as a_1 , but it enables the method to be extended easily to cases where there are two or more artificial variables - see later example.]

From (4), re-write $A = a_1$ as $A + x + y - s_3 = 4$ (5)

Re-labelling the rows:

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Referring to the Simplex tableau above:

1st row: new objective (1st stage of method)

2nd row: original objective (2nd stage of method)

Each stage of the method involves applying the ordinary Simplex method.

To minimise A: the positive coefficients of x & y mean that x or y could be increased (alternatively maximise -A).

Choose *x* as the pivot column (eg) and apply the ratio test:

3rd row: $\frac{12}{2} = 6$, 4th row: $\frac{30}{6} = 5$, 5th row: $\frac{4}{1} = 4$

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As A = 0, the 1st stage has been successfully completed.

Now remove the 1st row, and the columns for A and a_1

 $(a_1 \text{ is a non-basic variable and is being set to 0})$

Solution so far: x = 4, y = 0, P = 4

The 2nd stage is now to maximise P, as usual.

The remainder of the working is as follows:

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Choose s_3 as the pivot column, and apply the ratio test: 8th row: $\frac{4}{2} = 2$, 9th row: $\frac{6}{6} = 1$, 10th row: n/a

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Choose y as the pivot column, and apply the ratio test:

12th row:
$$\frac{2}{\left(\frac{4}{3}\right)} = \frac{3}{2}$$
, 13th row: n/a , 14th row: $\frac{5}{\left(\frac{5}{6}\right)} = 6$

The coefficients of $s_1 \& s_2$ in (15) are both positive, so we have maximised P.

Solution: $x = \frac{15}{4}, y = \frac{3}{2}, P = \frac{21}{4}$ (B)

(3) The Big M (Simplex) Method (same example)

This starts off in the same way as the 2-Stage method, by creating the artifical variable.

$$P - x - y = 0 \quad (1)$$

$$2x + 3y + s_1 = 12 \quad (2)$$

$$6x + 5y + s_2 = 30 \quad (3)$$

$$x + y - s_3 + a_1 = 4 \quad (4)$$

We now modify the objective to:

maximise $P = x + y - Ma_1$, where M is a large number (eg 1000)

This ensures that minimising a_1 is given 1st priority, as the Ma_1 term has the biggest effect on P.

Re-write as $P = x + y - M(4 - x - y + s_3)$ giving $P - (1 + M)x - (1 + M)y + Ms_3 = -4M$

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We now carry out the Simplex method as usual, and we should find that the RHS of the objective row becomes free of *M*.

Choose *x* as the pivot column (eg) and apply the ratio test:

2nd row:
$$\frac{12}{2} = 6$$
, 3rd row: $\frac{30}{6} = 5$, 4th row: $\frac{4}{1} = 4$ (as before)

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Once M only appears in the a_1 column, we can set a_1 to 0, and remove the a_1 column, arriving at the same tableau as at the end of the 1st stage of the 2-stage method (and then continue as before).

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(4) Complications for the Simplex Method

The following is a summary of the various ways in which complications can arise.

(i) Objective function parallel to a constraint line (if two variables) or plane (if three).

As for the Linear Programming method, more than one solution is possible.

(ii) When applying the 2-Stage Simplex or Big M method, it may not be possible to reduce a_1 to 0; ie there may not be a solution to the problem.

(iii) Artificial variables may be needed for more than one constraint. In this case, let $A = a_1 + a_2 + \cdots$ for the 2-Stage Simplex, and have $-M(a_1 + a_2 + \cdots)$ in place of $-Ma_1$ for the

Big M method.



$$P - x - y = 0 \quad (1)$$

$$2x + 3y - s_1 + a_1 = 12 \quad (2)$$

$$6x + 5y + s_2 = 30 \quad (3)$$

$$y - s_3 + a_2 = 5 \quad (4)$$

Minimise $A = a_1 + a_2 = (12 - 2x - 3y + s_1) + (5 - y + s_3)$ $\Rightarrow A + 2x + 4y - s_1 - s_3 = 17$

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Example (Big M):

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than as per the 2-stage simplex method

(iv) Constraints that are equalities

Replace with two inequality constraints:

ie for x + y = 4: replace with $x + y \le 4$ & $x + y \ge 4$

Example

Big m Method Maximise scry, subject to 23c+5512 and x=4 $2x+y+s_1 = 12$ $x_1 - s_2 + a_1 = 4$ $3c+s_3 = 4$ Modified objective : maximise $3c+y - Ma_1$ (P) $= 3c+y - M(4-x+s_2)$ $= (1+M)sc+y - Ms_2 - 4M$

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=) solly is: $s_1 = 4$, y = 4 ($s_1 = 0$, $s_2 = 0$, $s_3 = 0$) R = 8

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(v) x + y < 4 (eg)

Use $x + y \le 4$ instead, and reduce x or y slightly, if necessary.

(vi) Big M method: to minimise P = x + yModify to minimising $x + y + Ma_1$ (instead of maximising $x + y - Ma_1$)

(vii) If x (for example) can be negative, then replace x with $x_1 - x_2$, where $x_1, x_2 \ge 0$ (This allows x to be negative, if necessary.)