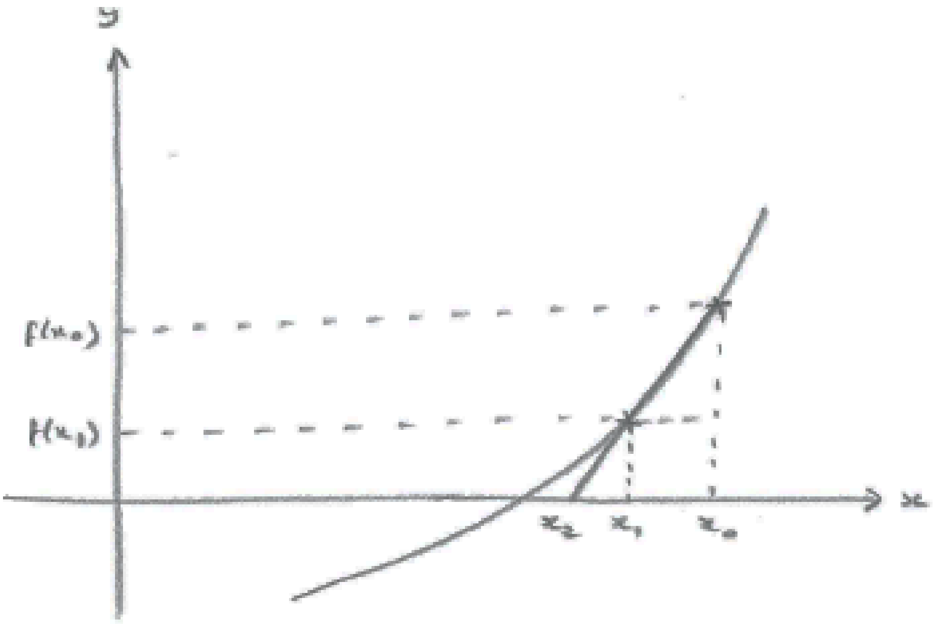
Numerical Solution of Equations - Secant method

(2 pages; 12/6/20)

(1) Referring to the diagram below, the Secant method formula is:

$x_2 = \frac{x_1 f(x_0) - x_0 f(x_1)}{f(x_0) - f(x_1)}$ (where x_0 & x_1 are two initial estimates).

[This is derived later on.]



In general, $x_n = \frac{x_{n-1} f(x_{n-2}) - x_{n-2} f(x_{n-1})}{f(x_{n-2}) - f(x_{n-1})}$

(2) The formula can be derived from the Newton-Raphson formula:

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ and $f'(x_1) \approx \frac{f(x_0) - f(x_1)}{x_0 - x_1}$

so that $x_2 = x_1 - \frac{f(x_1)}{\left(\frac{f(x_0)-f(x_1)}{x_0-x_1}\right)} = x_1 - \frac{f(x_1)(x_0-x_1)}{f(x_0)-f(x_1)}$

$= \frac{x_1 f(x_0) - x_1 f(x_1) - x_0 f(x_1) + x_1 f(x_1)}{f(x_0)-f(x_1)}$

$= \frac{x_1 f(x_0) - x_0 f(x_1)}{f(x_0)-f(x_1)}$

(3) The formula can also be derived by linear interpolation:

Consider the situation where x_2 lies between x_1 and x_0. Then x_2 is a weighted average of x_1 and x_0:

$$x_2 = \left(\frac{f(x_0)-f(x_2)}{f(x_0)-f(x_1)}\right) x_1 + \left(\frac{f(x_2)-f(x_1)}{f(x_0)-f(x_1)}\right) x_0$$

The formula will be valid for all positions of x_2, and since

$f(x_2) = 0$, $x_2 = \frac{x_1 f(x_0) - x_0 f(x_1)}{f(x_0)-f(x_1)}$

(4) Example: A spreadsheet can be used to find the root of

$x^3 - x - 1 = 0$, with $x_0 = 2$ and $x_1 = 1.5$

A6

=+(A4*B5-A5*B4)/(B5-B4)

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

	A	B	C	D	E	F
2	x(r)	f(x(r))				
3						
4	2	5.000000000				
5	1.5	0.875000000				
6	1.393939	0.314578290				
7	1.334405	0.041685543				
8	1.325311	0.002529955				
9	1.324723	0.000022667				
10	1.324718	0.000000013				