STEP 2&3 – Trigonometry

Solve $sin\theta = cos4\theta$ for $0 < \theta < \pi$

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Solution

$sin\theta = sin(\frac{\pi}{2} - 4\theta)$

Hence $\theta = \frac{\pi}{2} - 4\theta + 2n\pi$ (1) or $\theta = \left(\pi - \left[\frac{\pi}{2} - 4\theta\right]\right) + 2n\pi$ (2)

From (1), $5\theta = \frac{\pi(1+4n)}{2}$, so that $\theta = \frac{\pi(1+4n)}{10}$

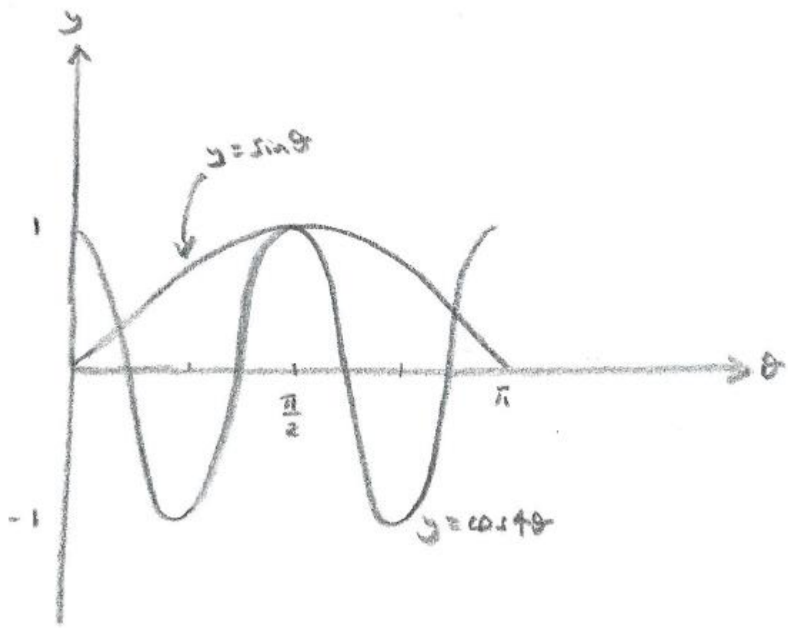
giving $\theta = \frac{\pi}{10}$, $\frac{\pi}{2}$ or $\frac{9\pi}{10}$

From (2), $-3\theta = \frac{\pi(1+4n)}{2}$, so that $\theta = \frac{-\pi(1+4n)}{6}$

giving $\theta = \frac{\pi}{2}$ again

Thus, the solutions are $\theta = \frac{\pi}{10}$, $\frac{\pi}{2}$ or $\frac{9\pi}{10}$

A sketch confirms that these are plausible.



Write $\sqrt{2(1 - cos\theta)}$ and $\sqrt{2(1 + cos\theta)}$ in the form $asin(b\theta)$ or $acos(b\theta)$

[Write $\sqrt{2(1-cos\theta)}$ and $\sqrt{2(1+cos\theta)}$ in the form $asin(b\theta)$ or $acos(b\theta)$]

Solution

$cos(2\theta) = cos^2\theta - sin^2\theta = 1 - 2sin^2\theta$,

so that $1 - cos(2\theta) = 2sin^2\theta$

and hence $\sqrt{2(1-cos\theta)} = 2\sin\left(\frac{\theta}{2}\right)$

Also, $cos(2\theta) = cos^2\theta - sin^2\theta = 2cos^2\theta - 1$,

so that $1 + cos(2\theta) = 2cos^2\theta$

and hence $\sqrt{2(1+cos\theta)} = 2\cos\left(\frac{\theta}{2}\right)$

Show that $cos^4\theta + sin^4\theta = 1 - \frac{1}{2}sin^2(2\theta)$

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Solution

Consider

$1 = (cos^2\theta + sin^2\theta)^2 = cos^4\theta + sin^4\theta + 2cos^2\theta sin^2\theta$

Then $cos^4\theta + sin^4\theta = 1 - 2cos^2\theta sin^2\theta = 1 - \frac{1}{2}(2cos\theta sin\theta)^2$

$= 1 - \frac{1}{2}sin^2(2\theta)$, as required.

What is the period of $2\,sin\left(3x+\frac{\pi}{4}\right)+3cos\,(\frac{2x}{3}-\frac{\pi}{3})$?

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Solution

The period T_1 of $2\,sin\left(3x+\frac{\pi}{4}\right)$ satisfies $3T_1 = 2\pi$

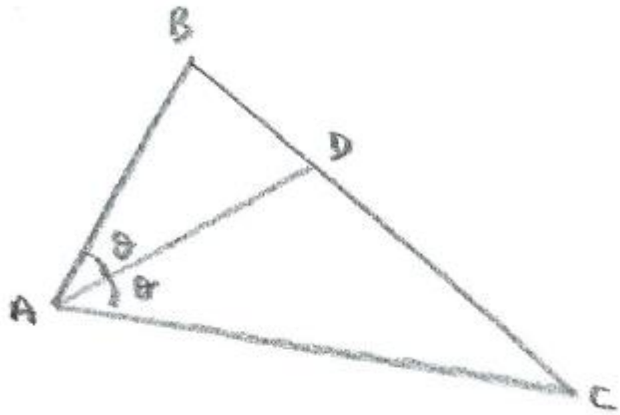
[as $2sin\left(3[0]+\frac{\pi}{4}\right) = 2sin\left(2\pi+\frac{\pi}{4}\right)$]; ie $T_1 = \frac{2\pi}{3}$

Similarly for $3cos\,(\frac{2x}{3}-\frac{\pi}{3}), \frac{2T_2}{3} = 2\pi$, so that $T_2 = 3\pi$

The period of the sum of these functions is the LCM of these two periods; ie 6π.

Angle Bisector Theorem

Referring to the diagram below, the Angle Bisector theorem says that $\frac{BD}{DC} = \frac{AB}{AC}$. Prove the Angle Bisector Theorem.



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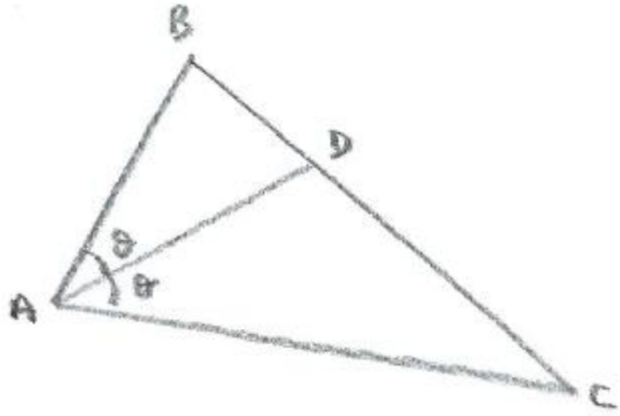

Solution

By the Sine rule for triangle ABD, $\frac{BD}{sin\theta} = \frac{AB}{sinADB}$ (1)

and, for triangle ADC, $\frac{DC}{sin\theta} = \frac{AC}{sinADC} = \frac{AC}{sinADB}$ (2)

Then (1) $\Rightarrow \frac{sin\theta}{sinADB} = \frac{BD}{AB}$ and (2) $\Rightarrow \frac{sin\theta}{sinADB} = \frac{DC}{AC}$

so that $\frac{BD}{AB} = \frac{DC}{AC}$

and hence $\frac{BD}{DC} = \frac{AB}{AC}$