STEP – Integration

How to convert from:

(i) $\int_0^\infty f(x)dx$ to $\int_\infty^0 g(x)dx$

(ii) $\int_0^a f(x)dx$ to $\int_a^0 g(x)dx$

Solution

(i) $\int_0^\infty f(x)dx$ to $\int_\infty^0 g(x)dx$

Let $u = \frac{1}{x}$

(ii) $\int_0^a f(x)dx$ to $\int_a^0 g(x)dx$

Let $u = a - x$

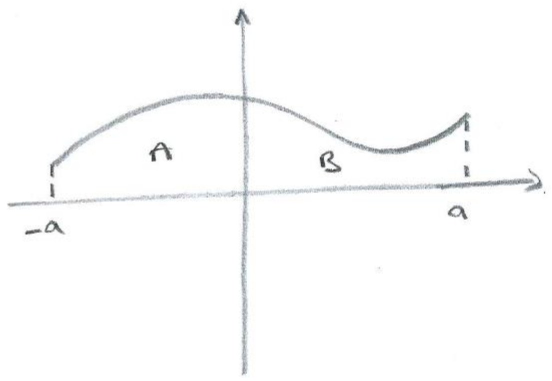
Simplify:

(i) $\int_{-a}^{a} f(-x)\,dx$

(ii) $\int_{0}^{a} f(\mathrm{a}-x)\,dx$

Solution

(i) $\int_{-a}^{a} f(-x)\, dx = \int_{-a}^{a} f(x)\, dx$



(ii) $\int_0^a f(\mathrm{a} - x)\, dx = \int_0^a f(x)\, dx$

[Let $u = a - x$, so that $du = -dx$ and

$\int_0^a f(\mathrm{a} - x)\, dx = \int_{\mathrm{a}}^{0} f(\mathrm{u})\, (-\mathrm{du}) = \int_0^a f(u)\, du = \int_0^a f(x)\, dx$]

$I = \int \sqrt{1 - x^2} dx$

Solution

(speculative substitution)

Let $x = sin\theta,$ so that $dx = cos\theta\ d\theta$

Then $I = \int cos\theta cos\theta d\theta = \frac{1}{2}\int 1 + cos2\theta\ d\theta$

$I = \int \tan x \ dx$

Solution

(integrating to find substitution)

$= \int \frac{\sin x}{\cos x} \; dx$

Integrating $sinx$ to give $-cosx$ reveals that the substitution

$u = cosx$ will work: $du = -sinx\, dx$,

so that $I = -\int \frac{1}{u} du = -lnu = -\ln(cosx) = \ln\,(secx)$

$I = \int sec^4\theta \; d\theta$

Solution

$I = \int sec^2\theta(1 + tan^2\theta)d\theta$

[Spotting that $\int sec^2\theta \; d\theta = tan\theta$] Let $u = tan\theta$,

so that $du = sec^2\theta \; d\theta$, and $I = \int 1 + u^2 \; du$

$I = \int \frac{1}{1+cosx}\ dx$

Solution

$I = \int \frac{1-cosx}{1-cos^2x}\ dx = \int \frac{1-cosx}{sin^2x}\ dx$

$= \int cosec^2x\, dx - \int \frac{cosx}{sin^2x}\ dx$ (and these can both be determined)

$I_n = \int_0^1 x^n e^{-x}\, dx$

Solution

(Recurrence relation)

Integrating e^{-x} and differentiating x^n gives:

$I_n = [\ -e^{-x}\, x^n\ ]_0^1 \ - \int_0^1 -nx^{n-1}e^{-x}\, dx$

$= \ -e^{-1} + 0 \ + nI_{n-1}$

Thus $I_n = nI_{n-1} - e^{-1}$