STEP/Integers: Exercises - Overview (19/11/23)

Q1

Can n^3 equal n + 12345670 (where *n* is a positive integer)?

Q2

Find all positive integer solutions of the equation

xy - 8x + 6y = 90

Q3

Show that $3^{57} - 2^{57}$ cannot be prime.

Q4

Prove that there are no positive integers m and n such that $m^2 = n^2 + 1$

Q5

Show that the product of 4 consecutive positive integers is never a perfect square.

Q6

Show that numbers of the form $4(n-1)^2 + 2$ can never be one more than a multiple of 3, where *n* is a positive integer.

Let h(a, b) denote the highest common factor of a & b. Suppose that b = ka + r, where k, a & r are positive integers.

Prove that h(a,b) = h(a,r).