STEP: Induction (2 pages; 2/7/25)

[See FM page for Notes & Exercises on ordinary Induction.]

Weak & Strong Induction

[$P(k)$ is the proposition that a particular result is true for $n = k$]

'Weak' induction is just the ordinary method

'Strong' induction is where we show that if $P(k-m)$,

$P(k-m+1), \ldots P(k)$ are correct, then $P(k+1)$ will be correct.
We then have to establish that $P(1), P(2), \ldots P(m+1)$ are correct.
(Weak induction corresponds to $m = 0$.)

Example: g_n is defined recursively as $(n^3 - 3n^2 + 2n)g_{n-3}$ for $n \geq 4$, and $g_1 = 1, g_2 = 2, g_3 = 6$

Show that $g_n = n!$ for $n \geq 1$

Solution

Assume that the result is true for $n = k-2, k-1$ & k.

Then $g_{k+1} = \left((k+1)^3 - 3(k+1)^2 + 2(k+1)\right)g_{k-2}$

$= (k+1)(k^2 + 2k + 1 - 3k - 3 + 2)(k-2)!$

$= (k+1)(k^2 - k)(k-2)!$

$= (k+1)k(k-1)(k-2)!$

$= (k+1)!$

So that the result is true for $n = k+1$ if it is true for

$n = k-2, k-1$ & k.

As it is true for $n = 1, 2$ & 3, it is therefore true for $n = 4, 5, \ldots$, and hence, by the principle of induction, it is true for all positive integers.