STEP – Differentiation

If $f(x) = x^2$, what is $f'(3x)$?

Solution

Method 1

Note that the differentiation is wrt $3x$ (rather than x).

Let $u = 3x$. Then $f'(3x) = f'(u) = \frac{d}{du}(u^2) = 2u = 2(3x) = 6x$

Method 2

$f'(x) = 2x \Rightarrow f'(3x) = 2(3x) = 6x$

Find $\frac{d}{dx}(x^x)$

Solution

Let $y = x^x$

Then $lny = xlnx$

and $\frac{1}{y}\frac{dy}{dx} = lnx + x(\frac{1}{x})$

so that $\frac{dy}{dx} = y(lnx + 1) = x^x(1 + lnx)$

Show that $\frac{d}{dx} log_a x = \frac{1}{xlna}$

Solution

$$\frac{d}{dx} log_a x = \frac{d}{dx}(log_a e . lnx) = \frac{1}{xlna} \text{ (as } log_a b = \frac{1}{log_b a})$$

Find $\frac{d}{dx}\left(x^{sinx}\right)$

Solution

$$\frac{d}{dx}\left(x^{sinx}\right) = \frac{d}{dx}\left(e^{lnx.sinx}\right) = e^{lnx.sinx}\left(\frac{1}{x}sinx + lnx.cosx\right)$$

$$= x^{sinx}\left(\frac{1}{x}sinx + lnx.cosx\right)$$