Continuous distributions

$\int_{-\infty}^{\infty} f(x)dx = 1$

$E(X) = \int_{-\infty}^{\infty} xf(x)dx$

Cumulative Distribution Function (CDF):

$F(x) = \int_{-\infty}^{x} f(t)dt \quad (F(-\infty) = 0;\ \ F(\infty) = 1)$

How can the median M be defined?

$\int_{-\infty}^{M} f(x)dx = \frac{1}{2}$

How can the mode m be defined?

$f'(x) = 0$ when $x = m$

Example

Consider the random variable X with pdf

$f_X(x) = 1$ for $0 \leq x \leq 1$, and 0 elsewhere

$[\int_0^1 1\, dx = [x]_0^1 = 1]$

The CDF of X is:

$F_X(x) = 0$ for $x < 0$,

x for $0 \leq x \leq 1$

1 for $x > 1$

To find the pdf of $Y = X^2$:

$F_Y(y) = P(Y \leq y) = P(X^2 \leq y) = P(X \leq \sqrt{y})$

$= F_X(\sqrt{y}) = \sqrt{y}$

Then the pdf of Y, $f_Y(y) = \frac{d}{dy} F_Y(y)$

$= \frac{d}{dy}\sqrt{y} = \frac{1}{2\sqrt{y}}$ (for $0 < y \leq 1$)

$[\int_0^1 \frac{1}{2\sqrt{y}}\, dy = [\sqrt{y}]_0^1 = 1]$

Note: Work with the CDF

Find $E(Y)$:

(a) from $\int_0^1 y\, f_Y(y) dy$

(b) from $\int_0^1 x^2\, f_X(x) dx$

Solution

(a) $E(Y) = \int_0^1 y\, f_Y(y) dy = \int_0^1 y \,.\, \frac{1}{2\sqrt{y}} dy$

$= \frac{1}{2}\int_0^1 \sqrt{y}\, dy = \frac{1}{2}\left[\frac{y^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right]_0^1 = \frac{1}{3}$

(b) $E(Y) = \int_0^1 x^2 f_X(x) dx = \int_0^1 x^2\, dx = \left[\frac{1}{3}x^3\right]_0^1 = \frac{1}{3}$