# STEP 2017, P1, Q1 - Solution (3 pages; 13/6/25)

**1** **(i)** Use the substitution $u = x\sin x + \cos x$ to find

$$\int \frac{x}{x\tan x + 1}\,\mathrm{d}x\,.$$

Find by means of a similar substitution, or otherwise,

$$\int \frac{x}{x\cot x - 1}\,\mathrm{d}x\,.$$

**(ii)** Use a substitution to find

$$\int \frac{x\sec^2 x\,\tan x}{x\sec^2 x - \tan x}\,\mathrm{d}x$$

and

$$\int \frac{x\sin x\cos x}{(x - \sin x\cos x)^2}\,\mathrm{d}x\,.$$

(i) **1st Part**

Let $u = xsinx + cosx$,

so that $du = (sinx + xcosx - sinx)dx = xcosx\ dx$

Then $\int \frac{x}{xtanx+1}\ dx = \int \frac{xcosx}{xsinx+cosx}\ dx$

$= \int \frac{1}{u}\ du = \ln|xsinx + cosx| + C$

**2nd Part**

$I = \int \frac{x}{xcotx-1}\ dx = \int \frac{xsinx}{xcosx-sinx}\ dx$

$\frac{d}{dx}(xcosx - sinx) = cosx - xsinx - cosx = -xsinx$

So let $u = xcosx - sinx$,

so that $I = -\int \frac{1}{u}\ du = -ln|xcosx - sinx| + D$

(ii) **1st Part**

[If necessary, try the simplest possible substitution. See comments later on.]

$\frac{d}{dx}(xsec^2x - tanx) = sec^2x + x\frac{d}{dx}(cosx)^{-2} - sec^2x$

$= -2x(cosx)^{-3}(-sinx) = 2xtanxsec^2x$

So let $u = xsec^2x - tanx$,

so that $I = \frac{1}{2}\int \frac{1}{u}\ du = \frac{1}{2}ln|\ xsec^2x - tanx| + E$

**2nd Part**

[As the 1st Part of (ii) turned out to be straightforward, it is likely that it is needed in some way for the 2nd Part.]

[The simplest approach for the 2nd part of (ii) is to convert it to the form $\int \frac{f\prime(x)}{(f(x))^2}dx$.]

As $\frac{xsinxcosx}{(x-sinxcosx)^2} = \frac{xtanxsec^2x}{(xsec^2x-tanx)^2}$,

let $u = xsec^2x - tanx$ again,

so that the integral $= \frac{1}{2}\int \frac{2xtanxsec^2x}{(xsec^2x-tanx)^2}dx = \frac{1}{2}\int \frac{1}{u^2}\ du$

$= -\frac{1}{u} + F = -\frac{1}{2(xsec^2x-tanx)} + F$