STEP 2016, Paper 2, Q9 – Solution (2 pages; 1/6/18)

(i) Accel. of bullet is $-\frac{R}{m}$

As this is constant, suvat eq'n '$v^2 = u^2 + 2as$' can be applied:

$0^2 = u^2 + 2\left(-\frac{R}{m}\right)a$ [a is the distance in this q'n]

so that $a = \frac{mu^2}{2R}$

Alternatively, by work-energy principle, gain in KE equals work done on bullet:

$0 - \frac{1}{2}mu^2 = -Ra$ etc

(ii) This q'n is a bit odd, in that the scenario has changed, and yet the resistance is the same. The only way that this would seem to make sense is if (for example) the material of the block is different in part (ii).

The MS refers to the scenario in (i) in the sol'n to (ii), which seems to complicate matters unnecessarily (especially given the comments above regarding the changed scenario).

By N3L, the block is subject to a force of R from the bullet (and as the surface is smooth, there are no other horizontal forces on it).

Let v be the speed of the block once the bullet has stopped inside the block. Note that the bullet is now travelling at speed v relative the table.

By conservation of momentum (relative to the table),

$mu = (m + M)v \quad (1)$

As the block has constant accel. $\frac{R}{M}$, applying '$v^2 = u^2 + 2as$':

$v^2 = 0^2 + 2\left(\frac{R}{M}\right)c \quad (2)$

For the bullet (relative to the table):

$$v^2 = u^2 + 2\left(-\frac{R}{M}\right)(b+c) \quad (3)$$

Using (1) to eliminate v,

$$(2)\&(3) \Rightarrow \frac{m^2u^2}{(m+M)^2} = \frac{2Rc}{M} = u^2 - \frac{2R}{m}(b+c)$$

Then using (4) to eliminate u^2,

$$\frac{m^2}{(m+M)^2}\left(\frac{2Ra}{m}\right) = \frac{2Rc}{M} = \frac{2Ra}{m} - \frac{2R}{m}(b+c)$$

$$\Rightarrow \frac{ma}{(m+M)^2} = \frac{c}{M} = \frac{a-b-c}{m}$$

$$\boxed{\Rightarrow c = \frac{mMa}{(m+M)^2}}$$

and $m\left(\frac{C}{M}\right) = a - b - c \Rightarrow b + c = a - \frac{m^2a}{(m+M)^2}$

$$\Rightarrow b = \frac{a(m+M)^2 - m^2a - mMa}{(m+M)^2} = \frac{a(M^2+mM)}{(m+M)^2} = \frac{aM}{m+M}$$

[there is a typo in the MS, which says $\frac{aM)}{(m+M)^2}$]