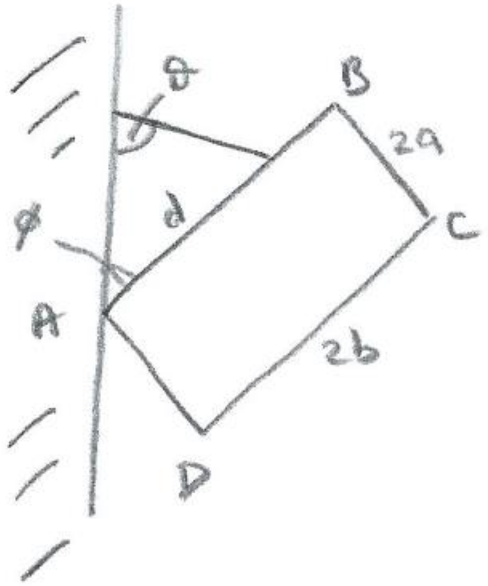
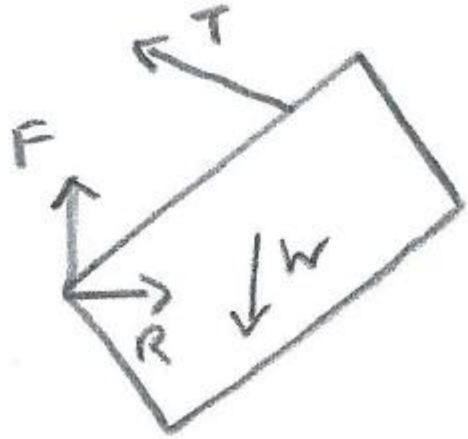
STEP 2014, P2, Q9 - Sol'n (2 pages; 18/6/20)

(i)





Referring to the diagrams:

Resolving horizontally, $R = Tsin\theta$ (1)

Resolving vertically, $F + Tcos\theta = W$ (2)

And $F = \mu R$, as equilibrium is limiting.   (3)

Component of W parallel to AD is $Wcos(90 - \phi)$

Then $M(A)$:

$Tsin(180 - \theta - \phi)d = Wcos(90 - \phi)b + Wsin(90 - \phi)a$ (4)

From (1), (2) & (3): $\mu Tsin\theta + Tcos\theta = W$ (5)

Then (4) ÷ (5) ⇒

$$\frac{sin(180-\theta-\phi)d}{\mu sin\theta + cos\theta} = cos(90 - \phi)b + sin(90 - \phi)a$$

$\Rightarrow dsin(\theta + \phi) = (cos\theta + \mu sin\theta)(acos\phi + bsin\phi)$, as required.

(ii) (2) becomes $-F + Tcos\theta = W$, leading to:

$$dsin(\theta + \phi) = (cos\theta - \mu sin\theta)(acos\phi + bsin\phi)$$

(iii) **1st part**

From (2), $F > 0 \Leftrightarrow W - Tcos\theta > 0 \Leftrightarrow \frac{W}{T} > cos\theta$

Also, from (4), $\frac{W}{T} = \frac{\sin(\theta+\phi)}{(acos\phi+bsin\phi)}$

So $\frac{\mathrm{d}\sin(\theta+\phi)}{(acos\phi+bsin\phi)} > cos\theta$

$\Leftrightarrow d > \frac{acos\theta cos\phi+bcos\theta sin\phi}{sin\theta cos\phi+cos\theta sin\phi} = \frac{a+btan\phi}{tan\theta+tan\phi}$

2nd part

The condition cannot be satisfied when $\frac{a+btan\phi}{tan\theta+tan\phi} > 2b$

$\Leftrightarrow a + btan\phi > 2b(tan\theta + tan\phi)$

$\Leftrightarrow a > b(2tan\theta + tan\phi)$, as required.