STEP 2014, P1, Q9 - Solution (2 pages; 13/11/19)

From $'v = u + at'$ vertically (upwards), $0 = Usin\theta + (-g)T_H$

$\Rightarrow T_H = \frac{Usin\theta}{g}$

From $'s = ut + \frac{1}{2}at^2'$, $0 = Usin\theta . T_L + \frac{1}{2}(-g){T_L}^2$

$\Rightarrow T_L = \frac{2Usin\theta}{g}$

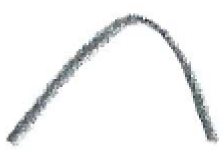
Note that $\frac{T_H}{T} = \frac{Usin\theta/g}{Ucos\theta/(kg)} = ktan\theta$

Also, $T = \frac{Ucos\theta}{kg} \Rightarrow kgT = Ucos\theta \Rightarrow Ucos\theta - kgT = 0$,

so that time T is when the horizontal component of the speed is zero; ie when the particle turns round horizontally.

When $ktan\theta < \frac{1}{2}$, $\frac{T_H}{T} < \frac{1}{2}$, so that $T > 2T_H = T_L$

ie the particle turns round horizontally after it has hit the ground (see sketch below)

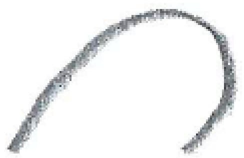


When $ktan\theta > 1$, $\frac{T_H}{T} > 1$, so that $T < T_H$

ie the particle turns round horizontally before it has reached its greatest height (see sketch below)

When $\frac{1}{2} < ktan\theta < 1, T < T_L$ and $T > T_H$

and the particle turns round horizontally after reaching its greatest height, but before it hits the ground (see sketch below)



[The remaining part is by no means obvious. We are expecting

$T = T_H$, as well as $T < T_L$]

When $ktan\theta = 1, k = \frac{cos\theta}{sin\theta}$,

so that $\frac{s_y}{s_x} = \frac{Usin\theta t - \frac{1}{2}gt^2}{Ucos\theta t - \frac{1}{2}kgt^2} = \frac{sin\theta(Usin\theta t - \frac{1}{2}gt^2)}{cos\theta(Usin\theta t - \frac{1}{2}gt^2)} = tan\theta,$

so that the particle continues to travel in a straight line, at an angle of θ to the horizontal, until it reaches its greatest height, when it returns along the same line (see sketch below)