STEP 2011, Paper 1 - Notes (6 pages; 11/6/18)

See separate documents for Sol'ns.

1	2	3	4	5	6	7	8
Sol'n	Sol'n	N	N	N	Sol'n	N	N

9	10	11		12	13
Sol'n	N	N		N	N

Q3 When manipulating sines and cosines – and with algebra generally - there is a danger of going round in circles, or at least of wasting time. The following method of 'forcing' an expression into the desired form can be applied:

eg to show that $\cos 2\theta + 2cos^2\theta = 2\cos 2\theta + 1$:

LHS $= 2\cos 2\theta + 1 + A$

where $A = -\cos 2\theta - 1 + 2cos^2\theta$

$= -(cos^2\theta - sin^2\theta) - 1 + 2cos^2\theta$

$= cos^2\theta + sin^2\theta - 1$

$= 0$

(It may well be the case that a more elegant method exists, but you could waste time by looking for it. Also, if you have made a mistake earlier on, this method should enable you to establish this reasonably quickly.)

Q4 There is a choice of using either the scalar product or the Cosine rule to find $cos\phi$. Generally the scalar product method is simpler, and here the form of the answer encourages it anyway.

As usual, it is possible to lose marks by failing to draw attention to the issue of avoiding division by zero (for example, pointing out that $p \neq 0$, so that 1/p is defined).

Q5 In theory, this question only requires standard methods, but it is complicated by some algebra and the commonly-occurring issue of having to justify division by some quantity, which mustn't be zero.

The official solution shows how the nature of the stationary point can be deduced by considering the value of I either side of the point, but not necessarily close to it - based on the fact that the stationary point is unique.

Q7 Differential equations seem to be an unpopular topic. In this case, the apparent length of the question was possibly a deterrent. In fact, there is nothing particularly complicated going on in this question. This topic could therefore be a good area in which to specialise and thereby gain an advantage over other candidates).

The solution of a differential equation can of course involve an awkward integration. In fact though, almost any substitution can be used for $\int \frac{1}{a-\sqrt{x}}dx$ (in addition to the two mentioned in the official solutions, $\sqrt{u} = a - \sqrt{x}$ is also possible).

Q8 Obviously for this type of question we need to be careful to consider all possibilities; in particular, negative values. However, a case by case approach can be avoided by use of the fact that $y = x^3$ is a (strictly) increasing function; ie $a^3 < b^3 \Leftrightarrow a < b$

Examiners are very keen on the use of $\Leftrightarrow$ (rather than a one-way argument). Unless indicated otherwise, it is normally safe to simply use the symbol ⇔ in place of ⇒ (provided that it is valid); ie it is sufficient to assert that each step is reversible.

The last part, in my view, was too obscure. Although it seems straightforward after reading the official solution, there are a number of possible approaches, involving several red herrings:

(a) using the result of part (i) by making a substitution of the form $n = q + a$ [doesn't work]

(b) showing that $q < p < q + 1$ unless some condition applies [it isn't true that p$> q$]

(c) showing that perhaps $q - a < p < q + b$ (or maybe

$p - c < q < p + d$) [the 1st approach turns out to work]

(d) showing that $q < p < q + 1$ unless some condition applies, with a further constraint to cover any cases where $q < p$ does not apply

[this seems rather obscure, but is in fact the main approach in the official solutions]

Although this question might have to be abandoned under exam conditions, the rule of "assume the simplest possible interpretation" probably applies (just about):

Here we might try (a) [because it's simpler to do than (b)], then (b) [but you could be forgiven for leaving the proof of $p > q$ until last; only to find that it isn't true]; but having found the drawback with (b) [and checking for any mistakes in applying both approaches (a) and (b)], the above rule would suggest modifying approach (b) to (hopefully) discover (d).

However, nothing is guaranteed in STEP, and applying the above rule in another question might lead nowhere!

Q10 It isn't always clear what aspects of a Mechanics question need to be explained. Here the official solution just states that the perfectly elastic bounce means that no energy is lost (rather than deducing this from the fact that $e = 1$).

Also, it isn't clear whether the condition "before the 2nd collision" (for the last part) is just intended to define the situation, or whether (as it turns out) candidates are supposed to show that the 2nd collision takes place after B reaches the top of its trajectory (which seems too straightforward for a STEP question).

Note how the expressions appearing in questions 9 and 10 appear equally complicated; yet question 9 involves much more algebra (ie you can't always tell that much from the look of a question).

Q11 This question obviously hinges on the standard result that the centre of gravity of an object will lie directly below any peg etc that it is hung from, if it is in equilibrium.

The official sol'n doesn't make much of the fact that PG bisects APB (it is relegated to Approach 4 for the last part only), but this would seem to make the trigonometry much more manageable. Since the peg is smooth, the two tensions are equal, and resolving horizontally shows that the angles APG and BPG are equal (as the bar + string system is in equilibrium). Denoting these equal angles by θ, the Sine rule then gives:

$\frac{3d}{sin\theta} = \frac{PG}{sin\alpha}$ and $\frac{4d}{sin\theta} = \frac{PG}{sin\beta}$, leading quickly to $3sin\alpha = 4sin\beta$

Apart from the above resolving of forces horizontally, it is possible to complete the question entirely using trigonometry (rather than resolving vertically and taking moments).

Q12 It's natural to expect that there should be some iterative shortcut for part (iii); ie that part (ii) can be invoked in some way. In fact this is only partly the case (for the two scenarios which start 12, as described in the official solution), and hardly saves much time. In fact, by the time you've thought about it and worked out how to do it, you will almost certainly end up taking longer.

The table method of presenting the different scenarios (as in the official solution) is perhaps easier to deal with than a tree diagram; but the tree diagram is probably easier to set up. So you could draw a tree diagram and then convert it into a table!

For the 2nd approach mentioned in part (ii), the probability of 121 is given as $\binom{m-1}{1}/\binom{m+2}{2}$: $\binom{m+2}{2}$ is the number of ways of choosing two positions for the £2 coins, out of the original m+2 positions, and $\binom{m-1}{1}$ is the number of ways of choosing a position for the remaining £2 coin, after 121 has been obtained (and there are $m-1$ coins left).

Q13 According to the Examiners' report, this question was not attempted by many of the stronger candidates. As it doesn't contain much in the way of theory, this could be a good topic to specialise in.

The only complication arises in (ii), where the logic has to be dealt with correctly. In the official sol'ns, in the alternative approach, the case of $d < 0$ is considered (which satisfies the required condition), but the case of $d > 0$ is not considered. If this is done, then a contradiction arises (which is what we want) - similar to the first approach. Since it is unlikely to be feasible to work with $arcsin\left(\frac{d}{2}\right)$, it makes sense to create an inequality involving $d = 0$, and fortunately this turns out to give the required contradiction.