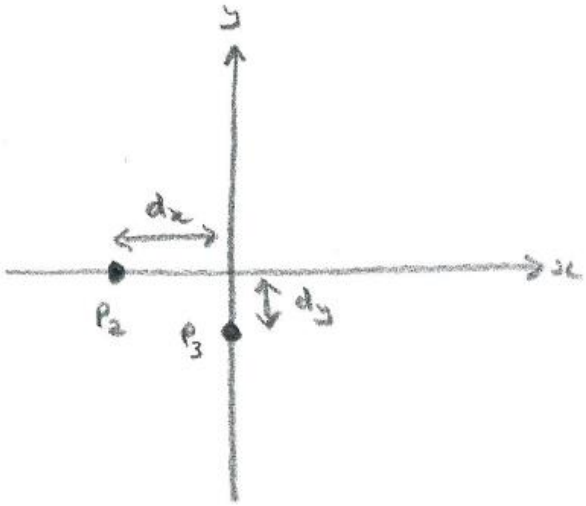
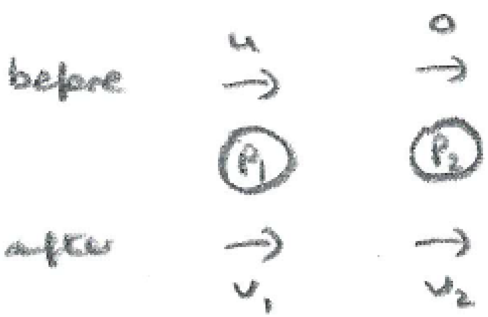
STEP 2009, Paper 2, Q10 – Solution (2 pages; 5/6/18)

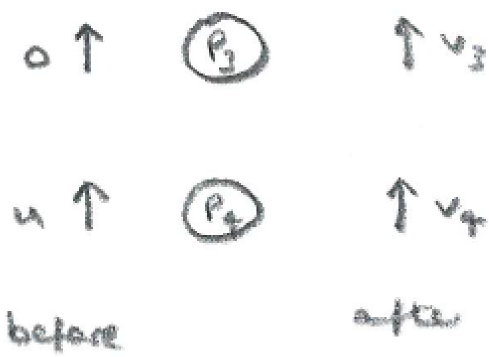
For this problem there are '8 degrees of freedom': 3 masses, 4 final velocities & the ratio of OP_3 to OP_2 initially (we could define the unit of mass as the mass of P_1, for example, without any loss of generality). However there will only be 6 equations: 2 from conservation of momentum, 2 from Newton's law of restitution, and 2 equating the times taken to reach O. So clearly something fortuitous must occur, if we are to deduce that the ratio of OP_3 to OP_2 initially is 1.

When faced with a large number of equations, look for any simple relations between variables (eg $v_1 = (1-\lambda)u_1$), that enable the number of variables (and equations) to be reduced quickly.

In this question, by delaying use of the conservation of momentum equations (involving the masses), we can actually stumble upon a very quick solution (quicker than the examiners presumably intended).







[Being unaware of the quick solution, it is natural to set up the conservation of momentum equations though.]

CoM: $m_1 u = m_1 v_1 + m_2 v_2$ (1)

NLR: $v_2 - v_1 = eu$ (2)

CoM: $m_4 u = m_4 v_4 + m_3 v_3$ (3)

NLR: $v_3 - v_4 = eu$ (4)

Collision of P_2 & P_3: time taken $= \frac{d_x}{v_2} = \frac{d_y}{v_3}$ (5)

Collision of P_1 & P_4: time taken $= \frac{d_x}{v_1} = \frac{d_y}{v_4}$ (6)

[Working with the simpler (5) & (6) initially:]

Let $\frac{d_x}{d_y} = \lambda$ [so we are trying to show that $\lambda = 1$]

Then from (5) & (6), $v_2 = \lambda v_3$ and $v_1 = \lambda v_4$

Considering (2) & (4) next, we can eliminate v_2 & v_1 to obtain

$\lambda v_3 - \lambda v_4 = eu$

Comparison with (4): $v_3 - v_4 = eu$ then reveals immediately that $\lambda = 1$, and hence that $d_x = d_y$, as required.