STEP 2009, Paper 1 - Notes (3 pages; 5/6/18)

See separate documents for Sol'ns.

1	2	3	4	5	6	7	8
Sol'n	Sol'n				N	N	

9	10	11		12	13	14
Sol'n		N		Sol'n		

Q6 Instructive, but fairly horrible.

At A Level, Integration methods can be grouped into 3 categories:

(a) substitution / 'by inspection' (if $\int f'(x).g(f(x)dx$, where $\int g(u)du$ can be found)

(b) Parts

(c) rearrangement (eg Partial Fractions)

Here, 'by inspection' isn't possible, but the substitution u = x+1 works.

The danger with Parts is that (as here) we could end up proving that I = I (where I is the integral to be found).

Apart from Partial Fractions, rearrangement is often neglected. It can be used here.

For (ii), we need the substitution u = 1/x, in order to keep the same limits (they are reversed, but we will obtain a factor of -1 when we differentiate u, and $\int_a^b f(x)dx) =$ -$\int_b^a f(x)dx)$. Also, the (x)s appear on the bottom, whereas the (u)s appear predominantly on the top, suggesting something of the form u=1/x.

For (iii), it is possible to find a & b s.t. the substitution u=ax+b converts the limits from 1 & 2 to ½ & 2, but (a) this is probably too obscure, and (b) it probably won't work (it doesn't, by the way).

The 1st official sol'n is not that satisfactory, as it requires you to start off without any obvious prospect of success, and the method really relies on a chance combination. However, it does illustrate the general idea of "if there's only one thing you can do, do that" – on the basis that the question has been designed to work.

The alternative method ignores the results established in the question (which isn't usually a good idea), but is probably the safest option in this case (with hindsight, of course).

There is a typo at the end of the official sol'n (it means $\int_1^2 \frac{x^5+x^3+1}{x^3(x+1)}$ dx, rather than $\int_1^2 \frac{x^5+3}{x^3(x+1)}$ dx).

Q7 The presence of m^2 in the answer is a clue that differentiation or integration might need to be carried out twice; thus suggesting integrating by parts twice.

For (ii), it is a fair bet that either an earlier result is to be used, or an earlier idea is to be modified or extended. The problem is to decide exactly what we are going to use. We can only really play around with the integrand e^xsin2xsin4xcosx until we recognise something from earlier on in the question. Conceivably we might have needed to go back to the result at the start of the question and derive something similar involving sines – though this turns out not to be the case.

Q11 This is really an algebra question.

A stress-free approach to (ii) is to use the result u+v+p+q=0 to give an expression for p+q; then use Newton's Law of Impact to give an expression for q-p; find p & q in terms of the other variables, and finally substitute for p and q in the CoM equation.

We also need to ensure that we are not dividing by zero at any point.

In general for this sort of question, we need to:

(i) decide which variables are to be eliminated

(ii) look for any shortcuts (eg involving difference of two squares)

(iii) make sure that we are using all the information (once only)

(iv) ideally, have a strategy where we can see our way to a suitable equation (which then just needs to be rearranged)

(v) at the end, 'force' expressions into the form appearing in the answer