STEP 2008, Paper 3 - Notes (4 pages; 13/5/20)

See separate documents for Sol'ns.

1	2	3	4	5	6	7	8
Sol'n	N	N	N	Sol'n	N	N	Sol'n

9	10	11		12	13
N	N			N	Sol'n

Q2 A short question (ie answer), by STEP standards.

The last parts (showing that the constant term is zero and that the sum of the coefficients is 1) is a bit of a trick question: having used induction to show that $S_k(n)$ is a polynomial of degree of k+1 in n, you could be forgiven for using induction for the last parts as well (as is possible), instead of just substituting for n = 0 and n = 1 in the definition.

Q3 Very straightforward (and short), and shows that STEP 3 questions are often easier than STEP 2: they just involve Further Maths (generally). In this case, very little needs to be known about ellipses though.

A bit of time can be saved in the last part by obtaining an expression for $(x + ae)^2$ in terms of y^2, rather than solving the equations for x and combining with the expression for y.

Surprisingly (as the examiners say), this question wasn't particularly popular (about a half of candidates attempted it), and apparently not done very well.

Q4 Hints:

For the 1st result in (i), consider the gradient at $y = 0$.

For the 2nd result in (ii), match it up with the 1st result.

For the 2nd result in (i), assume the simplest possible intended method; namely that y is to become $arcoshx$, and that $\tanh\left(\frac{y}{2}\right)$ is to become the RHS. Then $x = coshy$, and we just have to show that $\frac{coshy-1}{sinhy} = \tanh(\frac{y}{2})$.

The result in (ii) is rather unusual (using A>B to prove that A>2B).

For (iii): as usual, if in doubt, apply the method given in the previous part: so integrate the result in (ii) - (with limits of 1 & t).

Q6 As with most DE questions, the knowledge required is fairly limited: just integrating factors here.

There is a complication in this question, in that, by differentiating the original equation, some information is lost (that A=0 in $y = p^2 + 2xp + A$). Thus this original equation needs to be used, in order to establish the arbitrary constant at the end.

Q7 Hint: Consider the relation between eg b-c & a-c.

This question doesn't actually require much knowledge of complex numbers: just how to rotate by 60°.

Part (i) requires some experimentation (by drawing sample shapes). Symmetry can be invoked: the only plausible configurations are (a) all equilateral triangles are external (b) all internal (c) alternately external and internal (the actual one).

Part (ii) is straightforward (and easier than (i)); though involving a lot of algebra. As the Examiner's Report points out, it is possible to do it without having done (i).

Q9 Hint: As for most Mechanics questions, start with N2L.

To obtain the result in Part (i), it is possible to obtain an expression for v by integrating the acceleration and then using the boundary conditions to eliminate the constant of integration, and create the result. However, this can be combined into one step by setting the area under the acceleration-time graph between times T_0 and 1 to zero.

Q10 Hint: Assume that the standard results for tension and elastic PE can just be applied to each short string.

Only about 1 in 5 candidates tried this question. Given that (a) it is straightforward (b) the answer is short, and (c) there is a 'show that' part (which provides a check on the 1st part), this would be a very good question to have done (assuming you've covered Hooke's Law).

Q12 Predictably "only a handful of candidates attempted this" (with 3 strong attempts). Moment generating functions is another Statistics topic (along with certain distributions, such as the Poisson), where very little theory is needed, and the questions are usually fairly straightforward (by STEP standards).