STEP 2008, Paper 1 - Notes (3 pages;1/6/2018)

See separate documents for Sol'ns.

1	2	3	4	5	6	7	8
Sol'n	Sol'n	N			N		N

9	10	11		12	13	14
Sol'n	N	N			Sol'n	

Q3 The official solution to (ii) does seem rather obscure (especially for STEP 1), and not that 'similar' to (i). Presumably the following alternative argument wouldn't have been accepted (as it isn't similar to (i)):

The idea behind $\frac{x}{y}+\frac{y}{x}\geq 2$ (which is equivalent to $x^2+y^2\geq 2xy$) can be seen from an example, such as $\frac{2}{3}+\frac{3}{2}\geq 2$: $\frac{2}{3}$ and $\frac{3}{2}$ average at least 1, so $\frac{3}{2}$ exceeds 1 by at least as much as the amount that $\frac{2}{3}$ falls short of 1.

If $0<x\leq y$, rewrite $\frac{x}{y}+\frac{y}{x}\geq 2$ as $\frac{y}{x}-1\geq 1-\frac{x}{y}$, or $\frac{y-x}{x}\geq\frac{y-x}{y}$, which follows from $x\leq y$. By symmetry, the result also holds for $y\leq x$.

The same idea can be applied to $\frac{r}{s}+\frac{s}{t}+\frac{t}{r}\geq 3$, though there are now two cases to be considered: $0<r\leq s\leq t$ and $0<t\leq s\leq r$ (other cases are covered by re-labelling).

For the case where $r\leq s\leq t$, the required result is equivalent to

$\frac{t}{r}-1\geq\left(1-\frac{r}{s}\right)+(1-\frac{s}{t})$, or $\frac{t-r}{r}\geq\frac{s-r}{s}+\frac{t-s}{t}$

Then $r \leq s \Rightarrow \frac{s-r}{s} \leq \frac{s-r}{r}$ and $r \leq t \Rightarrow \frac{t-s}{t} \leq \frac{t-s}{r}$,

so that $\frac{s-r}{s} + \frac{t-s}{t} \leq \frac{s-r+t-s}{r} = \frac{t-r}{r}$, as required.

A similar argument works when $t \leq s \leq r$.

Q6 For the last part, consider $f(\frac{1}{2})$ and g(k).

Q8 Part (ii) can be tackled by exactly the same method as for part (i).

$\ln|x^2 - 1|$ was required, though the official sol'n bypasses this by using an unusual method. Probably for this reason, the Examiners' Report states that candidates weren't penalised for ignoring the modulus sign (though this isn't usually the case). In this question, it didn't affect the answer. But to deal with it you can consider the cases $|x|<1$, $|x|>1$ & $|x|=1$ separately. Because of the method they used, the official sol'n doesn't consider the case $|x|=1$.

Q10 The condition $V^2 sin^2\alpha > 2gh$ needs to be taken into account for full marks. (The Examiner's report mentions that very few candidates did this.)

Being a 'show that' question makes it low-risk.

Q11 The usual method(s) for equilibrium apply here (though fairly detailed work is required).

(Fairly obvious typo at the bottom of p29 of the official sol'ns: "from which it follows that $\phi = \mu$" should read "... $\phi = \lambda$")

Note the device of employing θ in the sol'n to the first part, using the definition of θ in the 2nd part of the question. If nothing else, it avoids the danger of defining θ to be some other angle in the 1st part.

If method 1 is adopted (resolving forces and taking moments), then the hard part of the question is being able to create and manipulate the equations in the time available. Although this makes it a bit risky, there is the compensation of it being entirely a "show that" question.

Notice that the question is only about the angles between the forces (since $\mu = tan\lambda$). For this reason, the three force result is sufficient. In itself, it only provides one piece of information (it reflects the fact that if one of the forces does not pass through the intersection of the other two forces, then there must be a non-zero moment; in order to determine the forces, it would be necessary to add the fact that the forces form a vector triangle (if an entirely geometrical approach was being followed); which is equivalent to resolving in two directions.