STEP 2005, Paper 2, Q11 - Solution (4 pages; 11/5/18)

[There are some discrepancies between the question and the H&A and Examiner's Report: The latter refers to a non-existent instruction in the question to set g to 10. In the H&A, the particle at A has become P (which is supposed to be the pulley). Sometimes questions are modified after having been sat, but this is usually mentioned.]

The 1st part is a standard A Level situation, but it is debatable whether the particles can be treated as being equivalent to a single unit (so that the tensions are internal forces), or whether this equivalence needs to be demonstrated algebraically. (One objection, for example, is that the accelerations of the two particles are in different directions.) The latter approach is obviously the safer one! (though the former approach did in fact appear in a recent A Level mark scheme (MEI, M2, June 2015)).

1st Approach (if allowed):

N2L on system of particles:

$m_2 g - m_1 g sin\alpha - \left(\frac{1}{2}\right) m_1 g cos\alpha = (m_1 + m_2)a,$

where $tan\alpha = 3/4$, so that $sin\alpha = \frac{3}{5}$ & $cos\alpha = 4/5$

and hence $a = \frac{m_2 g - m_1 g(\frac{3}{5}+\frac{2}{5})}{m_1+m_2} = \frac{(m_2 - m_1)g}{m_1+m_2}$, as required

2nd Approach

N2L for the particle at A: $T - m_1 g sin\alpha - \left(\frac{1}{2}\right) m_1 g cos\alpha = m_1 a$ (1)

N2L for the particle at B: $m_2 g - T = m_2 a$ (2)

Adding (1) & (2) then gives the equation in Approach 1.

[The remainder of the question involves applying the suvat equations.]

Let the following denote points on the inclined plane:

R: release, S: string breaks , M: maximum height reached

[It's very easy to confuse "release" with "breaking" in this question!]

Denote $\frac{(m_2-m_1)}{m_1+m_2}$ by k

Speed at S is $0 + kgT \quad [v = u + at]$

Let T_1 be the time after breaking that the maximum height is reached.

Then speed at M is $kgT - \left(gsin\alpha + \frac{1}{2}gcos\alpha\right)T_1 = 0$

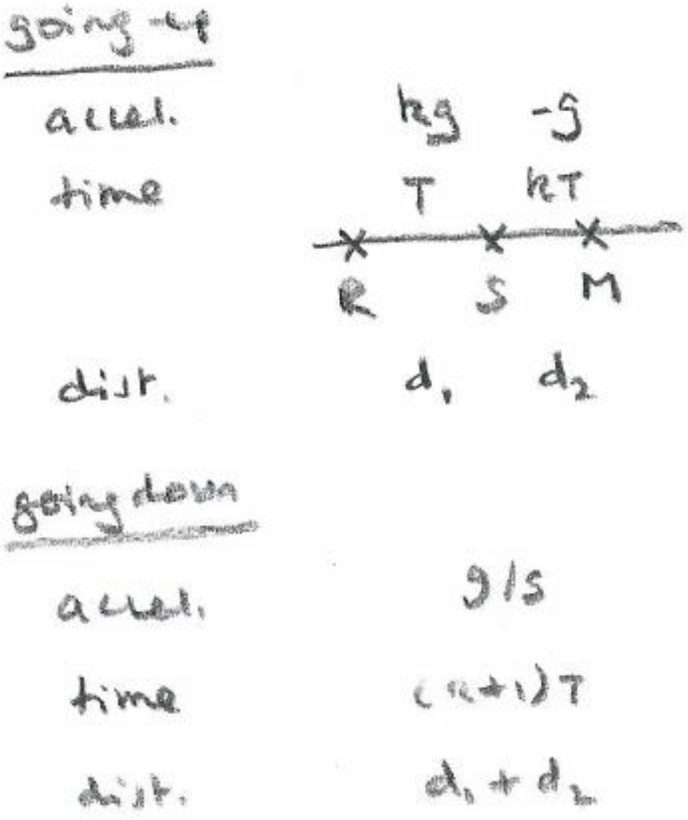
$\Rightarrow T_1 = \frac{kT}{\frac{3}{5}+(1/2)(4/5)} = kT,$

so that the time after release at which the particle at A reaches its maximum height is $T + kT = T\{\frac{(m_2-m_1)}{m_1+m_2}+\frac{(m_1+m_2)}{m_1+m_2}\} = \frac{2m_2T}{m_1+m_2}$

[A common pitfall with friction questions is to overlook the fact that the direction of friction reverses when the direction of motion is reversed.]

Going down, the acceleration is:

$gsin\alpha - \frac{1}{2}gcos\alpha = g\left(\frac{3}{5} - \left(\frac{1}{2}\right)\left(\frac{4}{5}\right)\right) = g/5$



Going up: $d_1 = \frac{1}{2}kgT^2$ & $d_2 = (kgT)kT - \frac{1}{2}g(kT)^2$

Going down: $d_1 + d_2 = \frac{1}{2}(\frac{g}{5})[(k+1)T]^2$

Hence $\frac{1}{2}kgT^2 + (kgT)kT - \frac{1}{2}g(kT)^2 = \frac{1}{2}(\frac{g}{5})[(k+1)T]^2$

so that $5k + 10k^2 - 5k^2 = (k+1)^2$

and $4k^2 + 3k - 1 = 0$; $(4k-1)(k+1) = 0$

Then, since $k + 1 \neq 0$ (as $(k+1)T$ is the time from R to M),

so that $\frac{(m_2 - m_1)}{m_1 + m_2} = 1/4$

Writing $\rho = \frac{m_1}{m_2}, \frac{1-\rho}{\rho+1} = 1/4$, so that $4 - 4\rho = \rho + 1$;

$5\rho = 3$, and $\frac{m_1}{m_2} = 3/5$

[Note: In the H&A, it is stated that the downwards acceleration is $\frac{g}{10}$: this should be g/5]