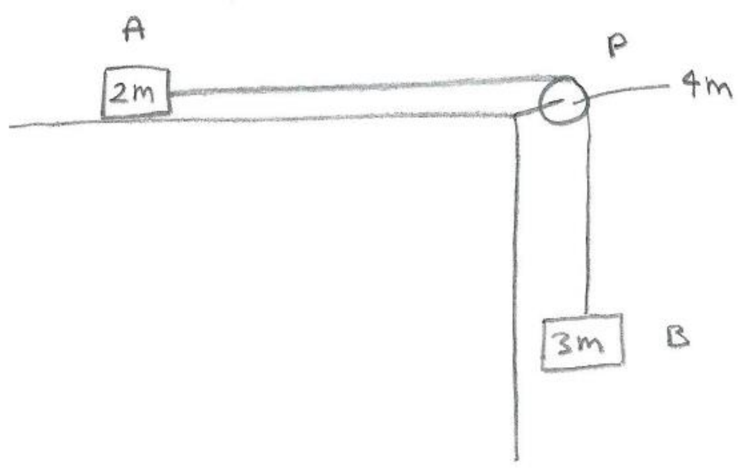
Rotating Bodies – Q1 [Problem/H] (20/6/21)



Initially block A is held at rest on a smooth table. The pulley P can rotate freely. The string leading from A to B, passing over P, is light and inextensible.

The pulley is a uniform disc of radius r, and the blocks can be modelled as particles.

Block A is released. The tension in the section of the string AP is T_A and in PB it is T_B.

Assuming that the string does not slip on the pulley, and that A does not reach P,

(i) Show that the angular acceleration of the pulley is $\frac{3g}{7r}$ $rads^{-2}$

(ii) Find T_A and T_B in terms of m and g.

Solution

(i) The moment of inertia, I of P is $\frac{1}{2}(4m)r^2 = 2mr^2$ [standard result for a disc about its axis]

The total moment of the external forces on P about its axis, $C = I\ddot{\theta}$, where $\ddot{\theta}$ is the angular acceleration of P.

$C = T_B r - T_A r$

Hence $r(T_B - T_A) = 2mr^2\ddot{\theta}$ (1)

The acceleration of A and B is $r\ddot{\theta}$ [the distance fallen by $B = r\theta$ (the arc length travelled by a point on the circumference of the pulley), and this is differentiated twice]

So, for A, N2L ⇒ $T_A = (2m)(r\ddot{\theta})$ (2)

and for B: $(3m)g - T_B = (3m)(r\ddot{\theta})$ (3)

Substituting for T_A and T_B from (2) & (3) into (1):

$$(3mg - 3mr\ddot{\theta}) - 2mr\ddot{\theta} = 2mr\ddot{\theta}$$

so that $7mr\ddot{\theta} = 3mg$, and $\ddot{\theta} = \frac{3g}{7r}$ $rads^{-2}$

Alternative method

By Conservation of energy,

$$\frac{1}{2}I(\dot{\theta})^2 + \frac{1}{2}(2m)(r\dot{\theta})^2 + \frac{1}{2}(3m)(r\dot{\theta})^2 - (3m)g(r\theta) = \text{constant}$$

(taking the initial position of B as the zero of PE; as before, $r\theta$ is the distance that B has fallen when P has rotated by θ rad)

Hence $\left\{\left(\frac{1}{2}\right)2mr^2 + \left(\frac{5}{2}\right)mr^2\right\}\left(\dot{\theta}\right)^2 - 3mgr\theta = \text{constant}$

Differentiating wrt time,

$\left(\frac{7}{2}\right)mr^2(2)\dot{\theta}\ddot{\theta} - 3mgr\dot{\theta} = 0,$

so that $7r\ddot{\theta} - 3g = 0,$ and $\ddot{\theta} = \frac{3g}{7r} \quad rads^{-2}$

(ii) From (2), $T_A = 2mr\left(\frac{3g}{7r}\right) = \frac{6mg}{7}$

From (3), $T_B = 3mg - 3mr\left(\frac{3g}{7r}\right) = \frac{12mg}{7}$