Rolling Wheel – Friction (6 pages; 15/2/21)

[Note: 'friction' is intended to mean the usual friction such as encountered by a block on a surface, rather than 'rolling friction', which is a type of resistance to motion.]

Contents

(A) Introduction

When a wheel rolls, its point of contact with the surface is stationary. [See "Rolling Wheel – Speed of point on circumference".] Effectively the wheel is continually toppling about the point of contact.

Consider the wheels of a bicycle (where there is rear wheel drive). The rear wheel can start rolling in two ways: either a torque is applied as the pedals are turned, whilst riding, or the bicycle is pushed by the handlebars whilst dismounted. (As a variant of the second way, if the wheel is allowed to roll down a slope, then the component of gravity down the slope takes the place of the pushing force.)

For the front wheel, only the second way is possible: either because the bicycle is pushed by the handlebars, or because the bicycle is being pedalled (but, as there is rear wheel drive, the front wheel is effectively being pushed along).

For both ways of initiating rolling, friction with the ground is taken advantage of; but, as will be seen, the direction of the frictional force is different in the two cases.

It will be seen that friction is only needed to accelerate (or decelerate) the wheel. If the wheel is moving at a constant speed, then there is no friction: this is because the point of the wheel that is in contact with the ground is stationary, and there is no force being applied (either as torque or as a pushing or pulling force) - as for a stationary block which is not being pushed or pulled. (Also, ignoring resistance forces, any friction would be applying a translational force to the wheel, and this cannot be the case, as it is moving with constant speed.)

The direction of the frictional force depends on whether the wheel is accelerating or decelerating, as well as whether the bicycle wheel is being pushed or pedalled.

However, this assumes a simple model where the wheel forms a perfect circle. In practice, the wheel will be deformed near the point of contact, and this will result in so-called 'rolling friction' (whether the wheel is moving at constant speed or accelerating/decelerating). Thus 'rolling friction' is a type of resistance to motion.

When the wheel is accelerating or decelerating, the friction does no work, as the point of contact with the surface is stationary – ie there is no displacement in the direction of the force. (In the case of 'rolling friction' however, the edge of the wheel is in prolonged contact with the surface, and so work is done against the resistance.) Thus the wheel does not lose energy on account of the frictional force.

If the surface isn't sufficiently rough, or if the torque or pushing force is too great, then slipping will occur. The frictional force will then be doing negative work, as the point of contact is now moving, and the frictional force will be acting to oppose this motion – ie in a direction opposite to the direction of travel.

(B) Wheel accelerating due to a torque

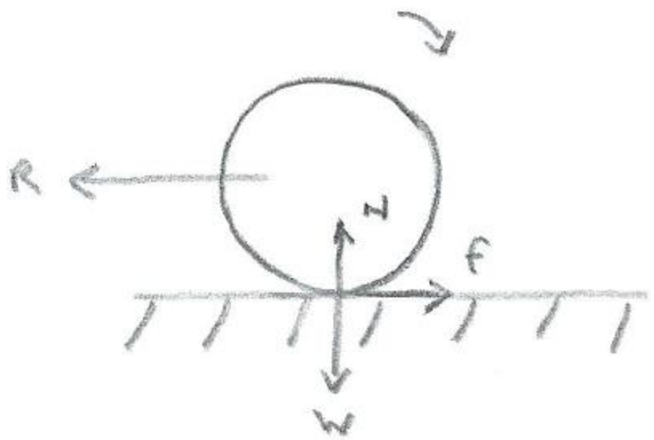

Figure 1: Wheel subject to a clockwise torque (it is rotating in a clockwise sense, and moving to the right)

Consider a stationary car that is suddenly accelerated by the application of a torque (or moment) to the wheels. If the acceleration is not too great and the road surface is sufficiently rough, then friction will be able to prevent the motion of the part of the wheel that is in contact with the road: This part of the wheel experiences a force to the left, as the torque is applied to the wheel. Friction opposes this, and is therefore to the right, preventing the contact point from moving.

Note that the friction involved here is static friction (with coefficient μ_s).

Thus the part of the wheel that is in contact with the road is at rest relative to the ground (the net effect of the motion forwards of the car and the opposite rotation of the wheel). This means that the wheel is rolling.

If instead the wheels slip (ie if the acceleration is too great, or if the surface is not rough enough), then there will be dynamic (or kinetic) friction at the point of contact (with coefficient μ_k; as if a block is sliding along a surface). μ_k is often significantly smaller than μ_s.

Referring to Figure 1, the translational acceleration of the wheel is $f - R$ (ie it doesn't arise directly from the driving force of the engine - although the driving force does give rise to the frictional force, which can be thought of as a means to convert rotational acceleration into translational acceleration). (The net rotational acceleration is zero, as the frictional force produces a torque that balances the torque of the engine.)

(C) Wheel decelerating due to a torque

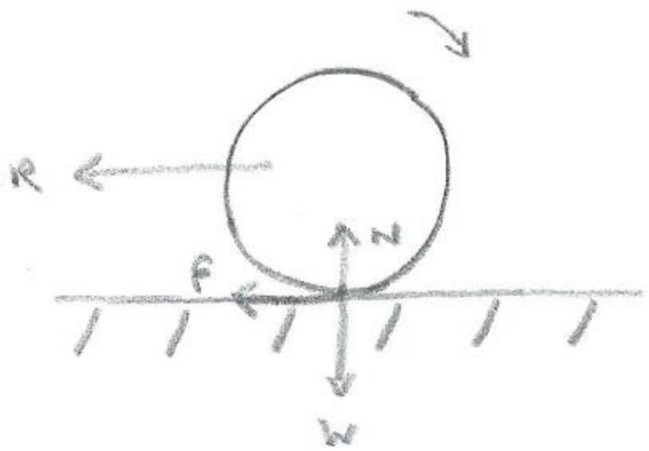

Figure 2: Wheel subject to an anti-clockwise (braking) torque (the wheel is still rotating in a clockwise sense, and moving to the right)

This could occur where a car or bicycle brakes, and attempts to make the part of the wheel that is in contact with the ground move to the right. Hence friction acts to the left (as it opposes attempted motion).

The translational acceleration is now $-f - R$ (ie a deceleration).

If limiting friction is exceeded, then the wheel slides. (See (E) Sliding.)

(D) Wheel rolling down a slope

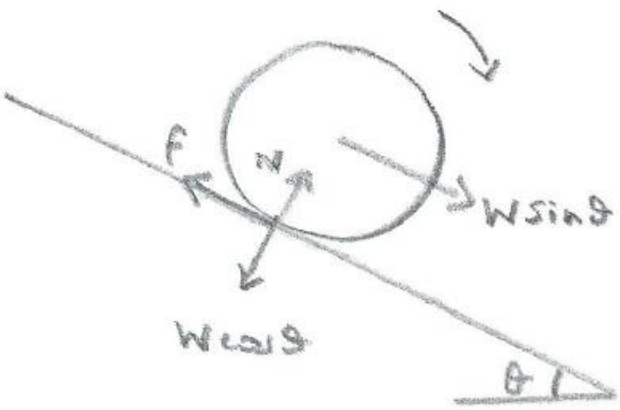

Figure 3: Wheel accelerating due to gravity (rotating in a clockwise sense, and moving to the right)

This is equivalent to the rear wheel of a (rear-wheel drive) bicycle being pushed by the handlebars whilst dismounted, or the front wheel of a bicycle in any situation (ie whether the bicycle is pushed or pedalled). The pushing force is replaced by the component of the weight of the wheel down the slope ($Wsin\theta$). The friction opposes the attempted motion, which is down the slope. So friction acts up the slope.

Note that various other permutations are possible: the wheel may be rolling up a slope, and there may also be a torque tending to accelerate or decelerate the wheel. In each case the direction of the frictional force will be determined by the directions and magnitudes of the torque and the component of the weight of the wheel. (Note that a wheel rolling up a slope is equivalent to a bicycle being slowed down by pulling on the handlebars whilst dismounted.)

(E) Slipping

When slipping occurs, there will be a torque imposed on the wheel by the (dynamic) frictional force. This increases the angular velocity ω of the wheel (if the wheel is moving to the right, the clockwise rotation will increase). Also, the effect of the frictional force is to reduce the translational velocity u of the wheel (all else being equal). Both of these effects tend to reduce the velocity of the point of contact with the ground, $u - \omega r$ (where r is the radius of the wheel), until it is once again zero, and the wheel rolls again. Prior to that the wheel is sliding and rotating, and is sometimes said to be 'sliding and rolling'. The term 'pure rolling' can be used to indicate that sliding has stopped.

This motion also applies to a ten-pin bowling ball, where the initial angular velocity may be zero, as the ball lands on the surface.

When a car (or bicycle) skids, the deceleration can be significantly reduced. The acceleration is still given by $-f - R$, as in (C), but the frictional force f is reduced, as the coefficient of friction will be the (usually) lower dynamic coefficient (compared with the static coefficient that applies when the wheel is rolling).