Rolling Wheel – Friction (11 pages; 3/5/23)

(1) Friction is normally thought of as a force providing resistance to the motion of an object (eg a block sliding on a surface). In the case of rolling objects, such as a wheel (assuming no slipping is involved), we will see that no work is done by friction (and hence no energy is lost to friction), if it is assumed that the wheel is perfectly circular.

However, in practice the wheel will be deformed near the point of contact, and because of prolonged contact with the surface negative work will be done by this so-called 'rolling friction', which is therefore a type of resistance to motion.

Any exam question which mentions friction encountered by a car or train etc. is usually referring to 'rolling friction' only – though (as will be seen below) ordinary friction also plays a crucial role in the motion.

(2) When a wheel rolls (assuming no slipping takes place), its point of contact with the surface is stationary. [See "Rolling Wheel – Speed of point on circumference".] Effectively the wheel is continually toppling about the point of contact.

(Note that it is also possible for there to be a combination of rolling and slipping – as when a ten-pin bowling ball is thrown, for example.)

Because the point of contact with the surface is stationary, ordinary friction (as opposed to 'rolling friction') can be investigated by considering the forces on an imaginary stationary block at the point of contact. Friction will act to oppose attempted motion of this block, arising from one or more forces. As will be seen, the direction of friction will depend on the circumstances of the situation.

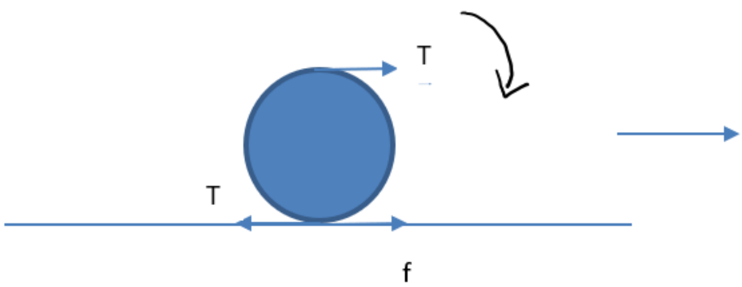
For the following examples, air resistance and rolling friction are being ignored.

(3) Example A: Tyre rolling on level ground at constant speed.

Here this is no force on the imaginary stationary block in the direction of motion (either translational or by way of torque), and so no friction is acting.

(4) Example B: Front-wheel drive car accelerating on level ground

Front Wheel



[Only forces affecting the motion are shown.]

There is a clockwise torque T, with the wheel rotating in a clockwise sense, and moving to the right.

The imaginary stationary block is subject to the force T to the left, and so a frictional force f acts to oppose this, to the right.

[Note that, in the usual case of a stationary block subject to a force T, the frictional force f would equal T, as the block is in equilibrium. But in the case of the rolling wheel, the situation is complicated by the fact that there is angular acceleration (the wheel is turning faster and faster). To find out the value of friction we need to set out the equations of motion.]

Equations can be set up as follows:

N2L applied to translational motion of the wheel:

$T - T + f = M\dot{v}$,

where v is the translational velocity (taking left to right as the positive direction) and M is the wheel's share of the mass of the car

Net moment of forces about centre of mass = Moment of Inertia (about centre of mass) × Angular acceleration

[angular equivalent of N2L]:

$\tau - rf = I_W\dot{\omega}$ (where torque $\tau = 2rT$),

where $\dot{\omega}$ is the angular acceleration (taking clockwise as the positive direction)

and $I_W = \frac{1}{2}\lambda M r^2$, for a disc of mass λM and radius r, about its axis (the wheel having mass λM, where $0 < \lambda < 1$)

Rolling condition: the point of contact of the wheel with the ground is stationary. The translational velocity of the point of contact has two components: v for the centre of mass, and $-\omega r$ due to rotation about the centre of mass; so that $v - \omega r = 0$, and differentiating: $\dot{v} = \dot{\omega} r$

So $\tau - rM\dot{v} = \frac{1}{2}\lambda M r^2 \frac{\dot{v}}{r}$; $\tau = (1 + \frac{\lambda}{2})Mr\dot{v}$ & $\dot{v} = \frac{\tau}{(1+\frac{\lambda}{2})Mr}$

Also, $f = M\dot{v} = \frac{\tau}{(1+\frac{\lambda}{2})r} = \frac{2rT}{(1+\frac{\lambda}{2})r} = \frac{2T}{(1+\frac{\lambda}{2})}$

When the wheel is accelerating (or decelerating), the friction does no work, as the point of contact with the surface is stationary – ie there is no displacement in the direction of the force.

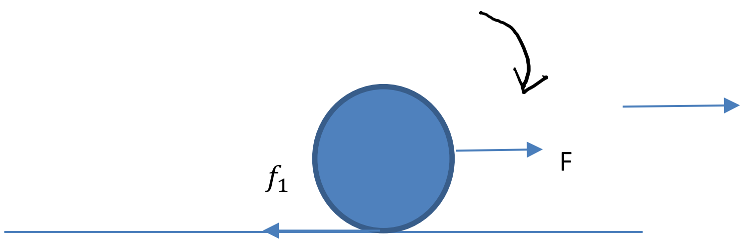
When the wheel is on the point of slipping, $f = \mu_s Mg$ (where μ_s is the static coefficient of friction).

Then $f = M\dot{v} \Rightarrow \mu_s Mg = M\dot{v} \Rightarrow \dot{v} = \mu_s g$ (ie this is the maximum acceleration possible).

With dynamic friction, the frictional force does negative work, as the point of contact is now moving, and the frictional force will be acting to oppose motion – ie acting in a direction opposite to the direction of travel. Thus, when the wheel is slipping, the frictional force is hindering the acceleration, instead of aiding it.

[This situation also applies to the rear wheel of a bike being pedalled.]

Rear wheel



For the rear wheel, there is no torque from the engine, but the wheel is pulled along by the chassis, and friction f_1 now acts to the left, to oppose the attempted motion.

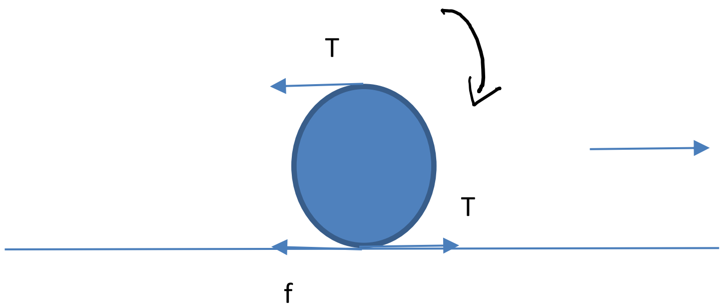
Now the equations are:

$F - f_1 = M\dot{v}$

$f_1 r = I_W \dot{\omega}$

[This situation also applies to the front wheel of a bike being pedalled.]

(5) Example C: Front-wheel drive car braking on level ground



The diagram shows the front wheel subject to an anti-clockwise torque, but still rotating in a clockwise sense, and moving to the right.

Assuming that there is no sliding, the imaginary stationary block would now be subject to a torque force T to the right, and the frictional force will now act to the left (opposing the torque force).

The equations of motion are:

$-T - f + T = M\dot{v}$

$-\tau + rf = I_W \dot{\omega}$

$\dot{v} = \dot{\omega}r$

So $-\tau + r(-M\dot{v}) = \frac{1}{2}\lambda M r^2 \frac{\dot{v}}{r}$; $\tau = -(1+\frac{\lambda}{2})Mr\dot{v}$ & $\dot{v} = -\frac{\tau}{(1+\frac{\lambda}{2})Mr}$

[Compare Examples B and C with the situations of (a) walking on a fairly firm surface, and speeding up, and (b) slowing down on the same surface. In the case of (a), friction is the only external force in the horizontal direction, and would therefore be expected to be in the direction of motion (the feet push down on the ground, and by N3L the ground pushes back on the feet: friction is the horizontal component of this reaction force). In the case of (b), friction is now helping the process of slowing down, and is opposite to the direction of motion.]

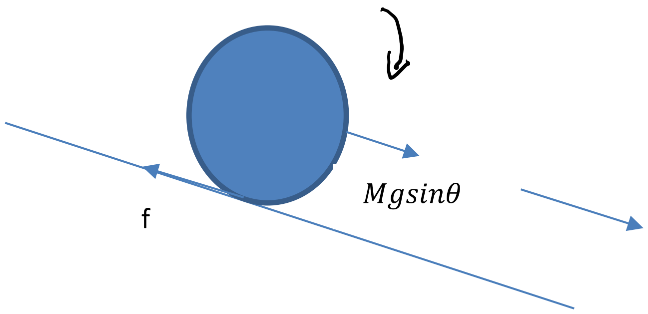
When the wheel is on the point of slipping,

$-f = M\dot{v} \Rightarrow -\mu_s Mg = M\dot{v} \Rightarrow \dot{v} = -\mu_s g$

So $\mu_s g$ is the maximum deceleration possible, and when the wheel starts to slides, so that $f = \mu_d Mg$ (where μ_d is the dynamic coefficient of friction), the maximum deceleration is the (usually) lower value of $\mu_d g$. Thus if slipping occurs whilst braking, the stopping distance is greater.

(6) Example D: Tyre rolling down a slope

[This is mathematically equivalent to the wheel of a bike that is being pushed by the handlebars, so that it accelerates - with the component of the weight down the slope taking the place of the pushing force.]



(where θ is the angle of the slope)

Assuming slipping doesn't occur, the imaginary stationary block is subject to the force $Mgsin\theta$ down the slope, and friction therefore opposes this force. (Once again, because of the rotational acceleration, $f \neq Mgsin\theta$.)

The equations of motion are:

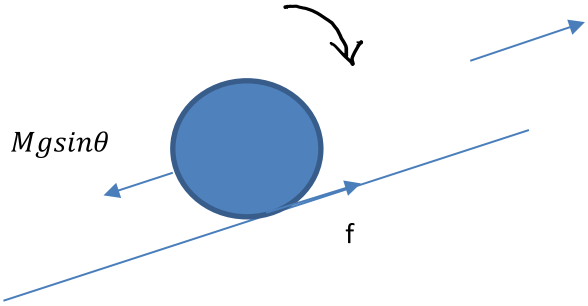
$$-f + Mgsin\theta = M\dot{v}$$

$$rf = I_W\dot{\omega}$$

$$\dot{v} = \dot{\omega}r$$

So $r(Mgsin\theta - M\dot{v}) = \frac{1}{2}Mr^2\frac{\dot{v}}{r}$; $gsin\theta - \dot{v} = \frac{1}{2}\dot{v}$ & $\dot{v} = \frac{2gsin\theta}{3}$

(7) Example E: Tyre rolling up a slope



[This is equivalent to the wheel of a bike that is being pulled by the handlebars, so that it decelerates.]

Again, the imaginary stationary block is subject to the force $Mgsin\theta$ down the slope, and friction opposes this force.

The equations of motion are:

$f - Mgsin\theta = M\dot{v}$

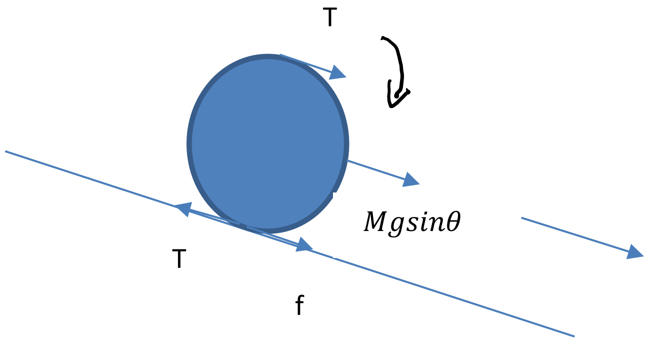
$-rf = I_W\dot{\omega}$

$\dot{v} = \dot{\omega}r$

So $-r(Mgsin\theta + M\dot{v}) = \frac{1}{2}Mr^2\frac{\dot{v}}{r}$; $-gsin\theta - \dot{v} = \frac{1}{2}\dot{v}$

& $\dot{v} = -\frac{2gsin\theta}{3}$

(8) Example F: Front-wheel drive car accelerating down a slope



The diagram show the front wheel (assumed to be rolling rather than slipping). There are now two forces on the imaginary stationary block (T & $Mgsin\theta$), in opposing directions. Supposing for the moment that the frictional force acts down the slope:

The equations of motion are:

$T + Mgsin\theta - T + f = M\dot{v}$

$\tau - rf = I_W\dot{\omega}$ (where $\tau = 2rT$)

$\dot{v} = \dot{\omega}r$

So $\tau - r(M\dot{v} - Mgsin\theta) = \frac{1}{2}\lambda Mr^2\frac{\dot{v}}{r}$;

$\tau + Mrgsin\theta = (1 + \frac{\lambda}{2})Mr\dot{v}$ & $\dot{v} = \frac{\tau}{(1+\frac{\lambda}{2})Mr} + \frac{1}{(1+\frac{\lambda}{2})}gsin\theta$

Now, $f = M\dot{v} - Mgsin\theta$, so that $f > 0$ when $\dot{v} > gsin\theta$;

ie when $\frac{\tau}{(1+\frac{\lambda}{2})Mr} + \frac{1}{(1+\frac{\lambda}{2})}gsin\theta > gsin\theta$,

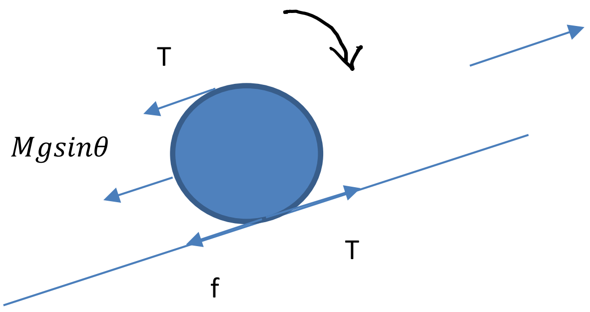
or $\frac{\tau}{(1+\frac{\lambda}{2})Mr} > (1 - \frac{1}{\left(1+\frac{\lambda}{2}\right)})gsin\theta$; ie $\tau > (\left(1 + \frac{\lambda}{2}\right) - 1)Mrgsin\theta$

ie $\tau > \frac{\lambda}{2} Mrgsin\theta$

ie the frictional force acts up the slope instead if $\tau < \frac{\lambda}{2} Mrgsin\theta$

(Consider the extreme case where $\tau = 0$. This is Example D, where friction acts up the slope.)

(9) Example G: Front-wheel drive car braking up a slope



The diagram shows the front wheel. Supposing again that the frictional force acts down the slope:

The equations of motion are:

$-T - Mgsin\theta - f + T = M\dot{v}$

$-\tau + rf = I_W\dot{\omega}$ (where $\tau = 2rT$)

$\dot{v} = \dot{\omega}r$

So $-\tau + r(-M\dot{v} - Mgsin\theta) = \frac{1}{2}\lambda Mr^2\frac{\dot{v}}{r}$;

$-\tau - Mrgsin\theta = (1 + \frac{\lambda}{2})Mr\dot{v}$ & $\dot{v} = -\frac{\tau}{\left(1+\frac{\lambda}{2}\right)Mr} - \frac{gsin\theta}{(1+\frac{\lambda}{2})}$

Now, $f = -M\dot{v} - Mgsin\theta$,

so that $f > 0$ when $\dot{v} < -gsin\theta$;

ie when $-\frac{\tau}{\left(1+\frac{\lambda}{2}\right)Mr} - \frac{gsin\theta}{(1+\frac{\lambda}{2})} < -gsin\theta$,

or $\frac{\tau}{(1+\frac{\lambda}{2})Mr} > (1 - \frac{1}{\left(1+\frac{\lambda}{2}\right)})gsin\theta$; ie $\tau > Mr(\left(1+\frac{\lambda}{2}\right) - 1)gsin\theta$

ie $\tau > \frac{\lambda}{2}Mrgsin\theta$

ie the frictional force acts up the slope instead if $\tau < \frac{\lambda}{2}Mrgsin\theta$

(Consider the extreme case where $\tau = 0$. This is Example E, where friction acts up the slope.)