Quadratics - Exercises (Sol'ns)(3 pages; 21/3/20)

(1**) Factorise $15x^2 + 34x + 16$

Solution

We want A and B such that $A + B = 34$ and $AB = (15)(16) = 240$

Again, the factorisation of 240 is $2^4 \times 3 \times 5$

Starting with |A| and |B| close to each other:

eg $A = 15,\ B = 16 \Rightarrow A + B = 31$

$A = 16,\ B = 15 \Rightarrow A + B = 31$ (ie no change)

$A = 20,\ B = 12 \Rightarrow A + B = 32$ (ie moving in the right direction)

$A = 24,\ B = 10 \Rightarrow A + B = 34$

Note: $A = 15, 12, 10$ also leads to a solution.

Then we have $(15x^2 + 24x) + (10x + 16\ )$

and $3x(5x + 8) + 2(5x + 8) = (3x + 2)(5x + 8)$

(2**) Derive the quadratic formula for the equation

$ax^2 + bx + c = 0$, by completing the square

Solution

First of all, $a\left(x^2 + \left(\frac{b}{a}\right)x + \frac{c}{a}\right) = 0$

$\rightarrow x^2 + \left(\frac{b}{a}\right)x + \frac{c}{a} = 0$

$\rightarrow (x + \frac{b}{2a})^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = 0$

$\rightarrow (x+\frac{b}{2a})^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a} = \frac{b^2-4ac}{4a^2}$

$\rightarrow (2ax+b)^2 = b^2 - 4ac$

$\rightarrow 2ax + b = \pm\sqrt{b^2-4ac}$

$\rightarrow x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

(3*) Find the turning points of the following quadratic functions (without differentiating)

(i) $y = x^2 + x - 2$

(ii) $s = 10t - 5t^2$

(iii) $s = 1 + 10t - 5t^2$

Solution

(i) Obtain roots from $x^2 + x - 2 = (x+2)(x-1)$

Then minimum point from either $x = \frac{1}{2}(-2+1)$, or completing the square: $x^2 + x - 2 = \left(x+\frac{1}{2}\right)^2 - \frac{1}{4} - 2$, to give $(-\frac{1}{2}, -\frac{9}{4})$.

(ii) Roots of 0 & 2; so maximum point when $t = 1$, to give (1,5); or completing the square:

$10t - 5t^2 = -5(t^2 - 2t) = -5(t-1)^2 + 5$

(iii) Alternative to above approach: maximum point when $t = 1$, as graph from (ii) is translated by $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, to give (1,6).

(4**) For what value of x does $(x+2)(x+4)$ have its minimum value?

Solution

Roots of $(x+2)(x+4)=0$ are $-2\ \&-4$, so minimum is at $x=-3$ (or complete the square, or find stationary point)

(5**) How to find k if $y=kx+1$ touches $y=x^2+2x+3$?

Solution

Any points of intersection occur where $kx+1=x^2+2x+3$;

ie $x^2+(2-k)x+2=0$

In order for the line to touch the curve, the discriminant must be zero;

ie $\Delta=(2-k)^2-4(2)=0$,

so that $k^2-4k-4=0$

Thus $k=\frac{4\pm\sqrt{16-(-16)}}{2}=2\pm2\sqrt{2}$

(6**) Give an example of a quadratic equation that has no real roots.

Solution

Anything of the form $(x+a)^2+b^2=0$ (where a & b are Real numbers, and $b\neq0$);

eg $(x+1)^2+1=x^2+2x+2=0$