

Quadratics - Exercises (10 pages; 3/2/26)

(1) Find the turning points of the following quadratic functions (without differentiating)

(i) $y = x^2 + x - 2$

(ii) $s = 10t - 5t^2$

(iii) $s = 1 + 10t - 5t^2$

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(i) $y = x^2 + x - 2$

(ii) $s = 10t - 5t^2$

(iii) $s = 1 + 10t - 5t^2$

Solution

(i) Obtain roots from $x^2 + x - 2 = (x + 2)(x - 1)$

Then minimum point from either $x = \frac{1}{2}(-2 + 1)$, or completing the square: $x^2 + x - 2 = \left(x + \frac{1}{2}\right)^2 - \frac{1}{4} - 2$, to give $\left(-\frac{1}{2}, -\frac{9}{4}\right)$.

(ii) Roots of 0 & 2; so maximum point when $t = 1$, to give (1,5); or completing the square:

$$10t - 5t^2 = -5(t^2 - 2t) = -5(t - 1)^2 + 5$$

(iii) Alternative to above approach: maximum point when $t = 1$, as graph from (ii) is translated by $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, to give (1,6).

(2) Factorise $15x^2 + 34x + 16$

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Solution

We want A and B such that $A + B = 34$ and $AB = (15)(16) = 240$

Again, the factorisation of 240 is $2^4 \times 3 \times 5$

Starting with |A| and |B| close to each other:

eg $A = 15, B = 16 \Rightarrow A + B = 31$

$A = 16, B = 15 \Rightarrow A + B = 31$ (ie no change)

$A = 20, B = 12 \Rightarrow A + B = 32$ (ie moving in the right direction)

$A = 24, B = 10 \Rightarrow A + B = 34$

Note: $A = 15, 12, 10$ also leads to a solution.

Then we have $(15x^2 + 24x) + (10x + 16)$

and $3x(5x + 8) + 2(5x + 8) = (3x + 2)(5x + 8)$

(3) Derive the quadratic formula for the equation

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Solution

First of all, $a \left(x^2 + \left(\frac{b}{a} \right) x + \frac{c}{a} \right) = 0$

$$\rightarrow x^2 + \left(\frac{b}{a} \right) x + \frac{c}{a} = 0$$

$$\rightarrow \left(x + \frac{b}{2a} \right)^2 - \left(\frac{b}{2a} \right)^2 + \frac{c}{a} = 0$$

$$\rightarrow \left(x + \frac{b}{2a} \right)^2 = \left(\frac{b}{2a} \right)^2 - \frac{c}{a} = \frac{b^2 - 4ac}{4a^2}$$

$$\rightarrow (2ax + b)^2 = b^2 - 4ac$$

$$\rightarrow 2ax + b = \pm \sqrt{b^2 - 4ac}$$

$$\rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(4) For what value of x does $(x + 2)(x + 4)$ have its minimum value?

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Solution

Roots of $(x + 2)(x + 4) = 0$ are -2 & -4 , so minimum is at $x = -3$ (or complete the square, or find stationary point)

(5) Find k if $y = kx + 1$ touches $y = x^2 + 2x + 3$

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Solution

Any points of intersection occur where $kx + 1 = x^2 + 2x + 3$;

$$\text{ie } x^2 + (2 - k)x + 2 = 0$$

In order for the line to touch the curve, the discriminant must be zero;

$$\text{ie } \Delta = (2 - k)^2 - 4(2) = 0,$$

$$\text{so that } k^2 - 4k - 4 = 0$$

$$\text{Thus } k = \frac{4 \pm \sqrt{16 - (-16)}}{2} = 2 \pm 2\sqrt{2}$$