

Pure - Miscellaneous: Exercises (3 pages; 24/3/20)**Contents**

(A) Indices

(B) Partial Fractions

(C) Recurrence relations

(Z) Miscellaneous

Key to difficulty:

* easier

** moderate

*** harder

(A) Indices(1*) (i) Does $\sqrt{4}$ equal 2 or ± 2 ? (ii) Simplify $\sqrt{x^2}$ (2*) Simplify $\left(1 + \left(1 + 2^{-\frac{1}{2}}\right)^{-1}\right)^{-1}$ **(B) Partial Fractions**(1***) Express $\frac{1}{(1-x^2)^2}$ in terms of partial fractions

(C) Recurrence relations

(1***) Consider the sequence defined by $u_n = au_{n-1} + b$, where a & b are real constants, and u_0 is given.

(i) What familiar sequences are special cases of this sequence?

(ii) Define a new sequence by $v_n = u_n + c$

For what value of c , in terms of a & b , will v_n be a geometric sequence?

For what value of a does this not work?

(iii) If $u_n = 2u_{n-1} + 3$, and $u_0 = 4$, find a formula for u_n in terms of n

(iv) Find a similar formula for $u_n = au_{n-1} + b$, where u_0 is given.

(v) Under what conditions will u_n be constant? Give a non-trivial example.

(Z) Miscellaneous

(1*) How are the following usually defined?

(a) Whole numbers (b) Natural numbers

(2**) Prove that $E' \Rightarrow L'$ is equivalent to $L \Rightarrow E$

(3*) What is a transcendental number?

(4***) Find the square roots of $49 - 12\sqrt{5}$

(5***) Show that $\sum_{r=0}^n \binom{n}{r} = 2^n$