Probability – Q2 [Practice/E] (23/5/21)

The events A and B are independent and are such that $P(A) = \frac{1}{5}$ and $P(A \cup B) = \frac{1}{2}$

Find:

(i) $P(B)$

(ii) $P(A' \cup B')$

(iii) $P(B' \cap A)$

(iv) $P(B'|A)$

Solution

(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

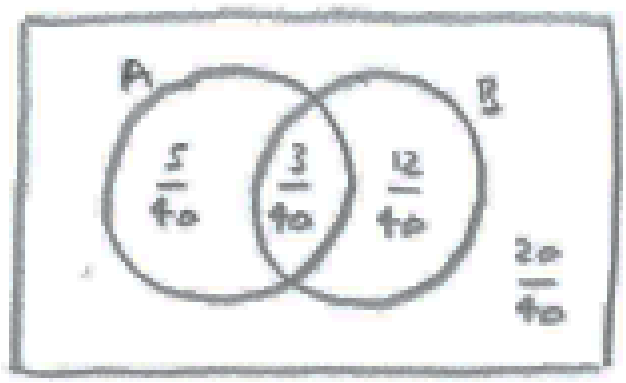
& $P(A \cap B) = P(A)P(B)$, from independence

Hence $\frac{1}{2} = \frac{1}{5} + P(B) - \frac{1}{5}P(B) \Rightarrow \left(\frac{1}{2} - \frac{1}{5}\right) = \frac{4}{5}P(B)$

$\Rightarrow P(B) = \frac{3}{10}\left(\frac{5}{4}\right) = \frac{3}{8}$

[At this point, a Venn diagram could be filled in,

using $P(A \cap B) = \frac{1}{5}P(B) = \frac{3}{40}$]



(ii) $P(A' \cup B') = P([A \cap B]') = 1 - P(A \cap B) = 1 - \frac{1}{5}P(B) = 1 - \frac{3}{40} = \frac{37}{40}$ (or from Venn diagram)

(iii) As A and B are independent, so are A and B'

[Independence of A and B ⇒ knowledge that A has occurred doesn't affect the probability of B occurring, or the probability of B not occurring]

Hence $P(B' \cap A) = P(B')P(A) = \big(1 - P(B)\big)P(A) = \frac{5}{8}\left(\frac{1}{5}\right) = \frac{1}{8}$

(or from Venn diagram)

(iv) As A and B' are independent, $P(B'|A) = P(B') = 1 - P(B) = \frac{5}{8}$

(or from Venn diagram; or $P(B'|A) = \frac{P(B' \cap A)}{P(A)} = \frac{\left(\frac{1}{8}\right)}{\left(\frac{1}{5}\right)} = \frac{5}{8}$)