

Permutations & Combinations – Exercises

(3 pages; 20/7/14) (*) denotes harder exercise

Summary of Possible Approaches

(a) Break down into various cases, and multiply by appropriate factors to take account of the number of possible arrangements for each case

(b) number of ways of allocating (eg) people to the 1st (eg) position $\times$ number of ways of allocating people to the 2nd position etc

(c) Find total including non-permissible cases and then deduct these cases.

(d) If items have to be next to each other, combine them into a single block of r items, and multiply by $r!$

(e) For r indistinguishable items, assume initially that they are different, and then remove duplication by dividing by $r!$ (or vice-versa)

(f) Listing of possible situations

(g) Use of a Venn diagram to deal with overlapping cases

(h) For probabilities, either (1) $\frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$

(assuming that the outcomes are equally likely), where combinations can often be used to find the numbers

or (2) $\text{Prob}(\text{eg } 1^{\text{st}} \text{ place being filled in a given way}) \times \text{Prob}(\text{2nd place being filled in a given way}) \times \dots$ (considering different cases separately, as in (a))

Exercises

(1) An exam candidate is supposed to answer 4 questions out of 5 in Section A, 3 out of 4 in Section B, and 2 out of 3 in Section C. If they don't read the instructions properly and answer 9 questions at random from the paper, what is the probability that they answer the questions they are supposed to?

(2) (i) 3 different sweets are to be shared amongst 5 children. In how many ways can this be done, if no child is to receive all 3 sweets?

(ii) What is the answer if the 3 sweets are indistinguishable?

(3) When choosing the venue for an international conference, 3 countries are shortlisted at random from a list of 9, of which 4 are European and 5 are from the rest of the world. What is the probability that at least 2 of the countries shortlisted are European?

(4) The following books are on a bookshelf: 4 novels, 3 history books, 2 biographies and 1 dictionary. In how many ways can they be arranged if the novels have to be together, and similarly for the history books and biographies?

(5) 3 bananas, 4 apples and 5 oranges are to be arranged in a row. In how many ways can this be done, assuming that the bananas etc are indistinguishable?

(6) (*) 2 boys and 3 girls are to sit in a row. How many arrangements are there in which the 2 boys are not next to each other and the 3 girls are also not next to each other?

(7)(*) 6 people (labelled A-F) are to be seated round a circular table. How many seating arrangements are possible if B and E are not to sit next to each other?