Parametric Equations (7 pages; 5/5/21)

Methods for converting from parametric to Cartesian form

(a) Make t the subject of one of the equations for x or y, and substitute for t in the other equation.

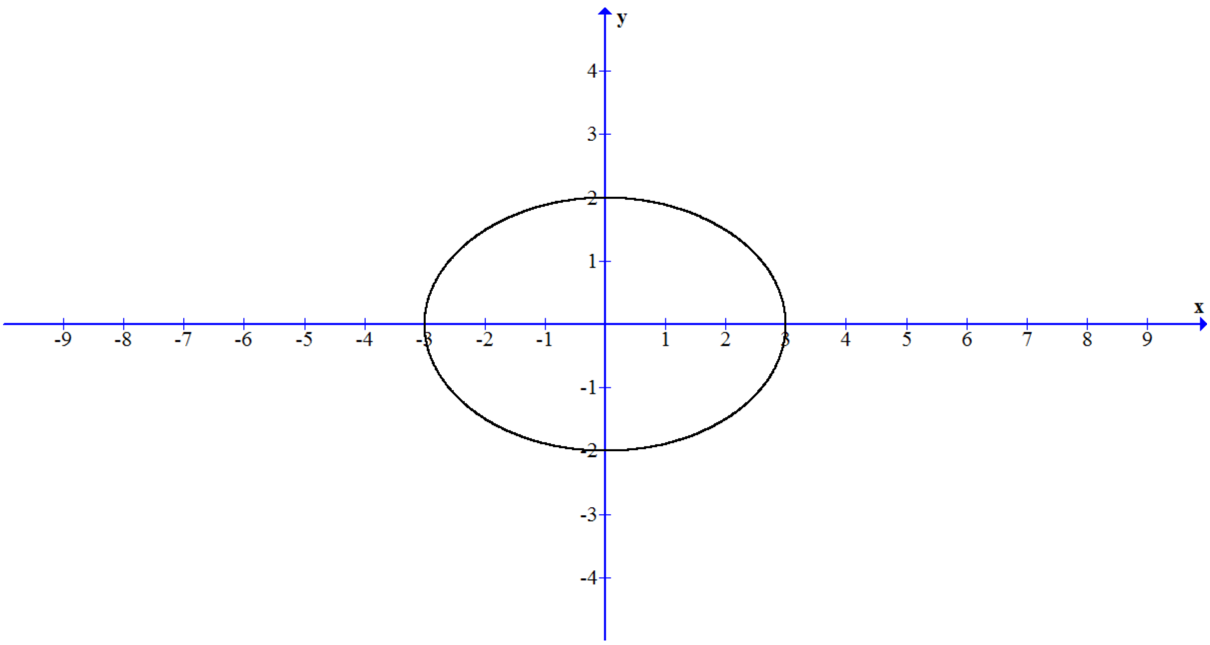
(b) Combine the equations for x & y in some way, so as to make t the subject (as in (i)).

(c) Make $f(t)$ the subject of both of the equations for x & y, and equate the two expressions (as in (ii), with $f(t) = t^2$), leaving perhaps a single t in the resulting equation.

Examples

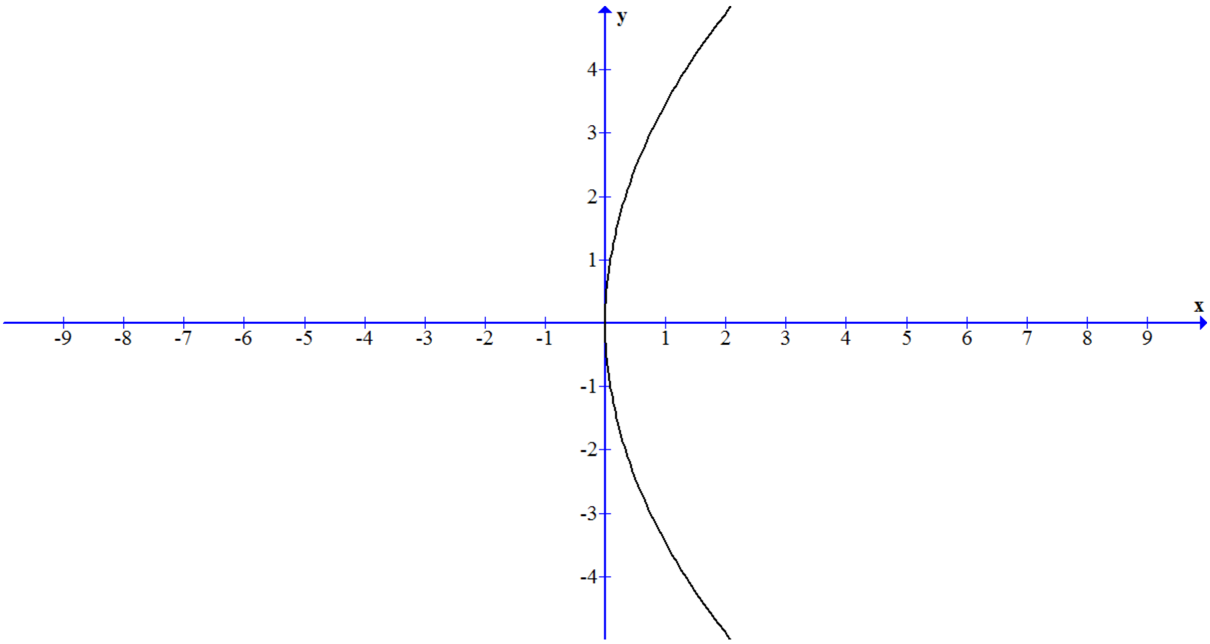
(i) $x = 3cos\theta,\ y = 2sin\theta$

Cartesian form: $\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$ (ellipse)



(ii) $x = 3t^2,\ y = 6t$

Cartesian form: $y^2 = 12x = 4(3)x$; parabola with focus at (3,0)



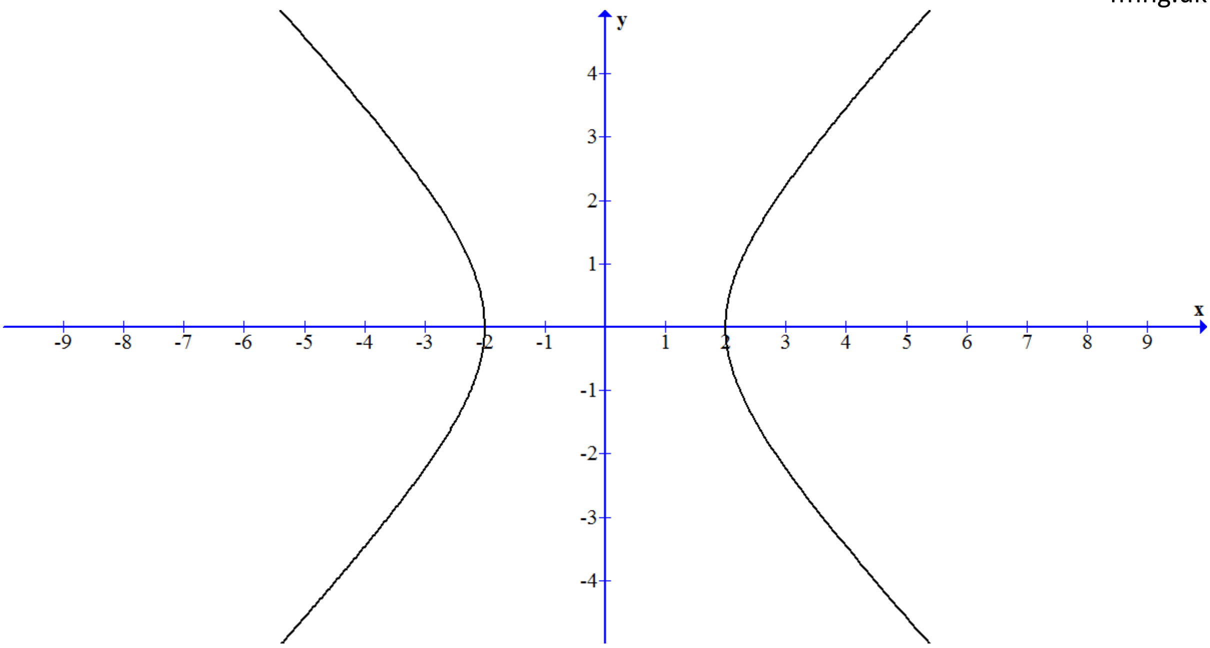
(iii) $x = t + \frac{1}{t}\ ,\ y = t - \frac{1}{t}$

Cartesian form:

$x + y = 2t; x - y = \frac{2}{t}$

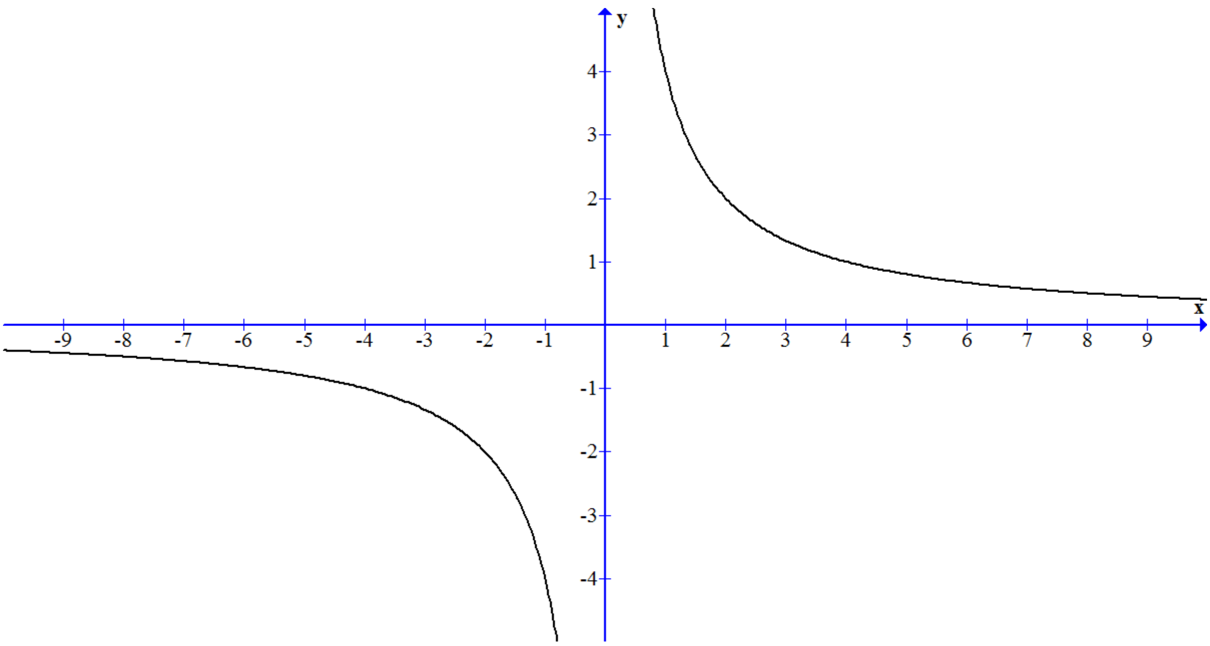
$\Rightarrow (x + y)(x - y) = 4$

$\Rightarrow \frac{x^2}{2^2} - \frac{y^2}{2^2} = 1$ (rectangular hyperbola)



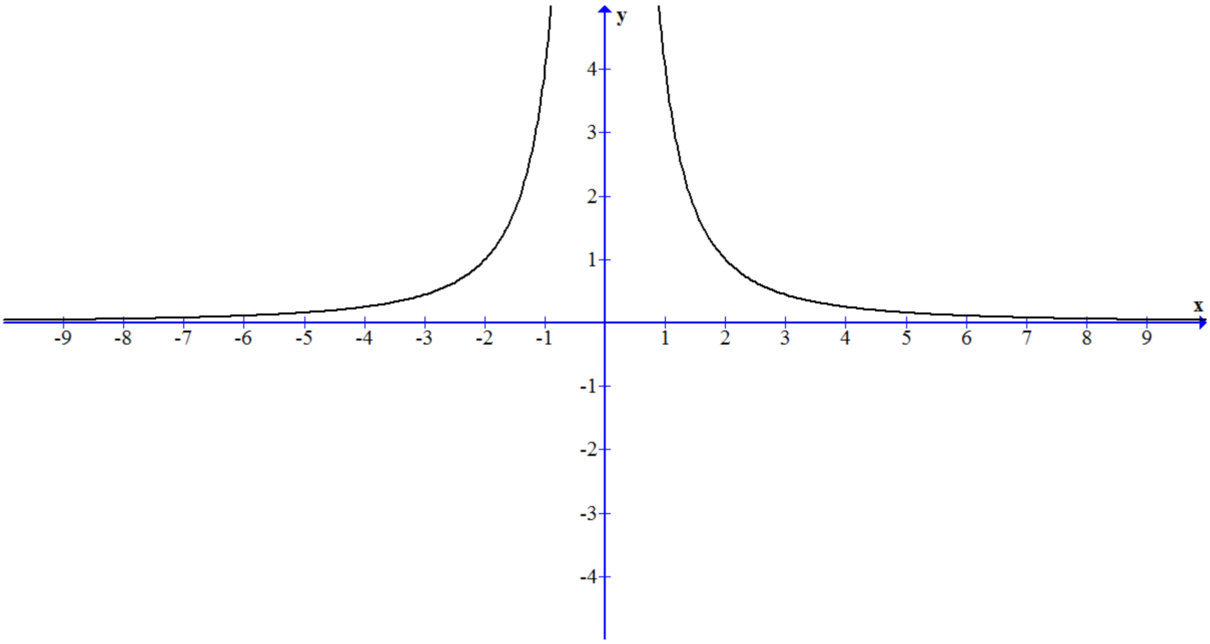
(iv) $x = 2t,\ y = \frac{2}{t}$

Cartesian form: $xy = 4$ (rectangular hyperbola, with asymptotes being x and y axes)



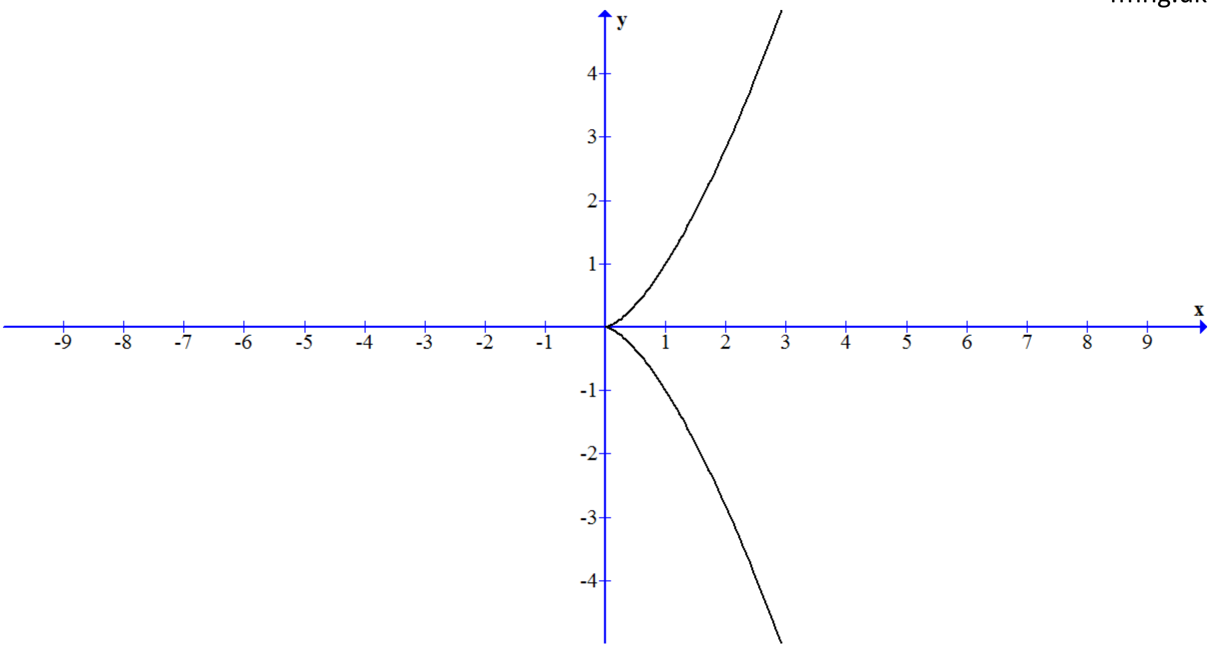
(v) $x = 2t,\ y = \frac{1}{t^2}$

Cartesian form: $x^2 = \frac{4}{y} \Rightarrow y = \frac{4}{x^2}$



(vi) $x = t^2,\ y = t^3$

Cartesian form: $x^3 = y^2$

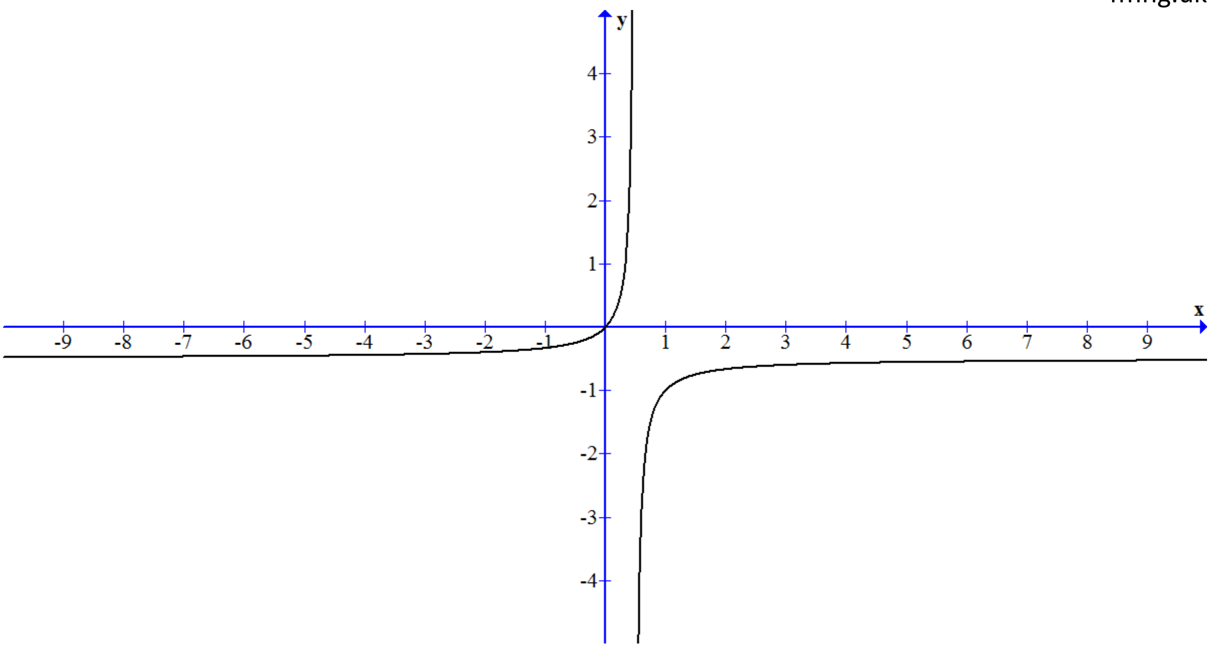


(vii) $x = \frac{t}{1+t}$, $y = \frac{t}{1-t}$

Cartesian form: $xy = \frac{t^2}{1-t^2}$; $y - x = \frac{2t^2}{1-t^2}$

$\Rightarrow y - x = 2xy \Rightarrow y(1 - 2x) = x$

$\Rightarrow y = \frac{x}{1-2x}$

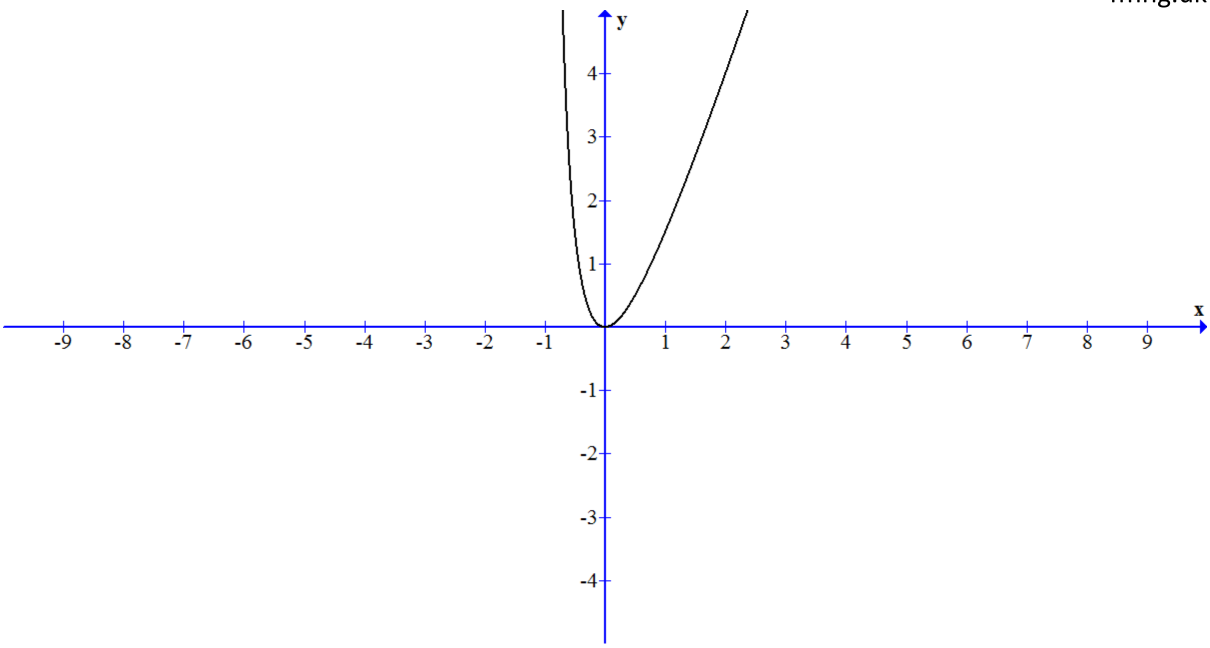


(viii) $x = \frac{t}{3-t}$, $y = \frac{t^2}{3-t}$

Cartesian form: $\frac{y}{x} = t \Rightarrow x = \frac{\left(\frac{y}{x}\right)}{3-\frac{y}{x}} = \frac{y}{3x-y}$

$\Rightarrow 3x^2 - xy = y \Rightarrow y(1 + x) = 3x^2$

$\Longrightarrow y = \frac{3x^2}{1+x}$


y
4
3
2
1
-1
-2
-3
-4
x
-9
-8
-7
-6
-5
-4
-3
-2
-1
1
2
3
4
5
6
7
8
9