PGF - Overview (12/6/21)

Q1 [Problem/H]

Given that $X_1, X_2, \dots, X_N$ & N are independent random variables, where the X_i are all distributed as X, and that

$S_N = X_1 + X_2 + \cdots + X_N$,

prove that $Var(S_N) = E(N)Var(X) + Var(N)[E(X)]^2$

The following results may be used:

(A) $E(X) = G_X'(1)$

(B) $VarX = G_X''(1) + G_X'(1) - [G_X'(1)]^2$

(C) $G_{S_N}(s) = G_N\big(G_X(s)\big)$

(D) $E(S_N) = E(N)E(X)$

Q2 [Problem/H]

A hen lays N eggs, where $N \sim P_o(\lambda)$, and each egg has probability p of hatching. Using any results about probability generating functions , show that the total number of eggs that hatch $\sim P_o(\lambda p)$

['Poisson hen']