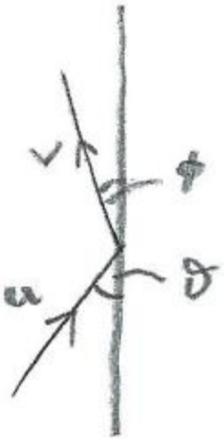
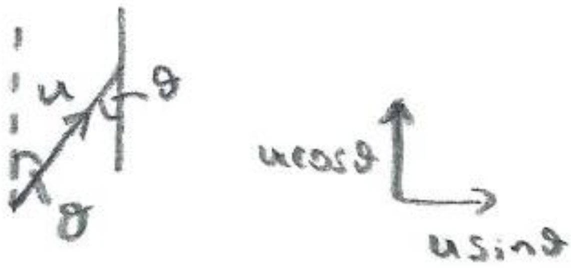
Oblique Impacts (4 pages; 23/8/18)

(1) Ball hitting wall





(i) motions in the two perpendicular directions (along the wall and perpendicular to it) are independent - as always in Mechanics (eg projectiles)

(ii) perpendicular to the wall: usual Newton's Law of Restitution applies:

$vsin\phi = eusin\theta$ (1)

(iii) along the wall: there is no frictional force opposing the ball (assuming that the wall is smooth), and so no impulse on it in that direction, and hence no change in momentum and therefore velocity

So $vcos\phi = ucos\theta$ (2)

[Note: This is not a situation where $e = 1$, as Newton's Law of Restitution isn't applicable. The coeff. of restitution measures the amount of bounce between two objects (eg a ball and a wall) in a head-on collision.]

(iv) Dividing (1) by (2), $tan\phi = etan\theta$

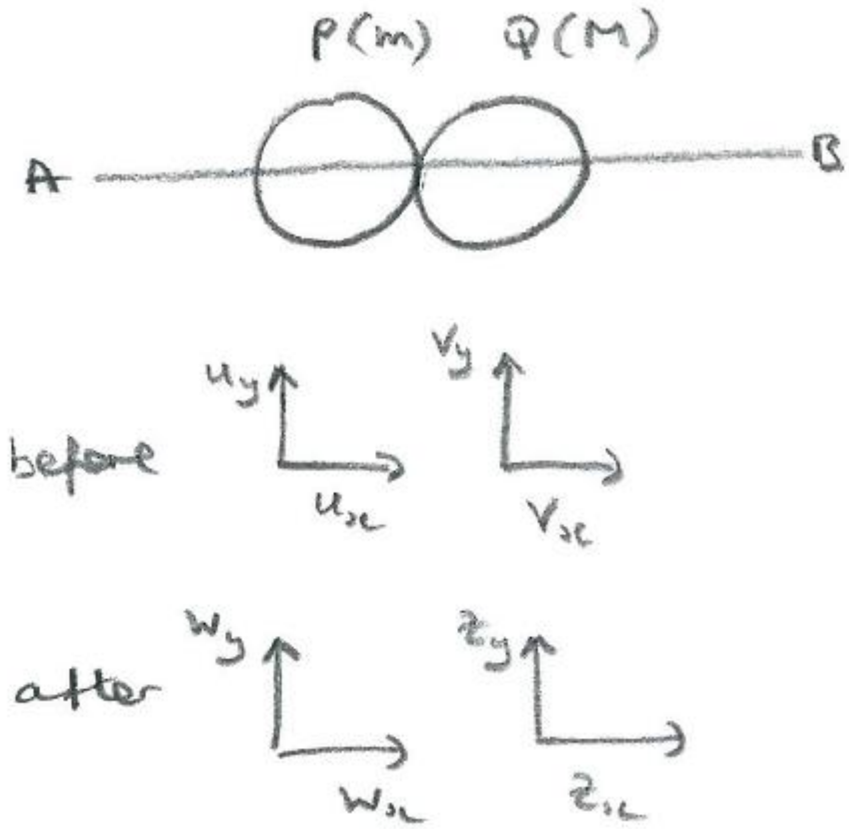
And squaring (1) & (2), and adding gives:

$v^2(sin^2\phi + cos^2\phi) = u^2(e^2sin^2\theta + cos^2\theta)$

so that $v = u\sqrt{e^2sin^2\theta + cos^2\theta}$

[this isn't a standard result to be quoted though]

(2) Two balls colliding (not head-on)



(i) motions in the two perpendicular directions (along AB and perpendicular to it) will be independent

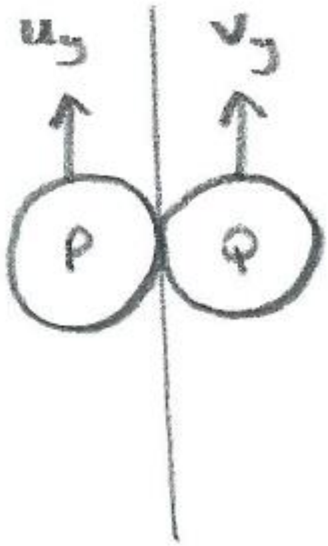
(ii) along AB, usual Conservation of Momentum and Newton's Law of Restitution apply:

$mu_x + Mv_x = mw_x + Mz_x$

$z_x - w_x = e(u_x - v_x)$

(iii) perpendicular to AB, the motions of P and Q are independent of each other (assuming the balls are smooth, so that there is no friction), as though they were on two sides of a thin (smooth) wall, where the 'upward' component of the velocity of each of them stays the same (ie as in (1))

ie $w_y = u_y$ and $z_y = v_y$



(iv) As in (1), Newton's Law of Restitution isn't applicable for the 'upward' motion. [However, the calculations could be done as though e were -1. This is the (impossible) situation where the two balls pass through each other without a reduction in speed. It wouldn't be a recognised approach for exam purposes.]

(v) 2D impulse-momentum eq'ns can be set up for P and Q:

For P: $\binom{-I}{0} = m\binom{w_x}{w_y} - m\binom{u_x}{u_y}$

For Q: $\begin{pmatrix} I \\ 0 \end{pmatrix} = M\begin{pmatrix} z_x \\ z_y \end{pmatrix} - M\begin{pmatrix} v_x \\ v_y \end{pmatrix}$

[If $w_x < u_x$ then P experiences an impulse in the negative x direction; ie a negative impulse $(-I)$ in the x direction.]

(giving $mu_x + Mv_x = mw_x + Mz_x$ and $w_y = u_y$ & $z_y = v_y$)

[Had P and Q been particles, instead of smooth spheres, then the impulse-momentum eq'ns would have been:

For P: $\begin{pmatrix} -I_x \\ -I_y \end{pmatrix} = m\begin{pmatrix} w_x \\ w_y \end{pmatrix} - m\begin{pmatrix} u_x \\ u_y \end{pmatrix}$

For Q: $\begin{pmatrix} I_x \\ I_y \end{pmatrix} = M\begin{pmatrix} z_x \\ z_y \end{pmatrix} - M\begin{pmatrix} v_x \\ v_y \end{pmatrix}$

(and Newton's Law of Restitution isn't applicable for particles)]