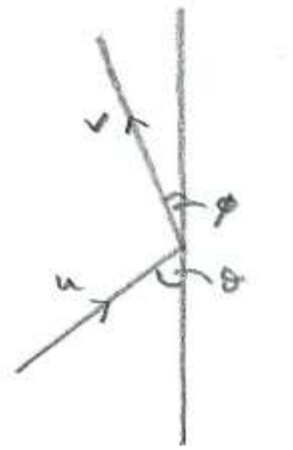
Oblique impact of ball with plane – Exercises

(10 pages; 5/5/25)



[Note: The following theory wouldn't apply to a particle, as it would not be capable of compression, and so Newton's law of restitution wouldn't apply.]

Referring to the diagram above, find an expression for $tan\phi$ in terms of $tan\theta$ and e.

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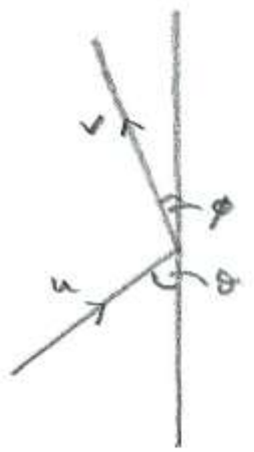
Solution

$vcos\phi = ucos\theta$ (A) and $vsin\phi = esin\theta$ (B)

Dividing (B) by (A), $etan\theta = tan\phi$



Find an expression for v in terms of $u, \theta\ and\ e$

(i) involving $cos\theta\ and\ sin\theta$

(ii) involving $tan\theta$

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Solution

(i) $(A)^2 + (B)^2 \Rightarrow v^2(cos^2\phi + sin^2\phi) = u^2(cos^2\theta + e^2sin^2\theta)$,

so that $v = u\sqrt{cos^2\theta + e^2sin^2\theta}$

(ii) $v = u\sqrt{cos^2\theta + e^2sin^2\theta} = u\sqrt{\frac{1+e^2tan^2\theta}{sec^2\theta}}$

$= u\sqrt{\frac{1+e^2tan^2\theta}{1+tan^2\theta}}$

When $\theta = 60°$ and $e = \frac{1}{\sqrt{3}}$, find ϕ, and v (in terms of u).

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Solution

When $\theta = 60°$ and $e = \frac{1}{\sqrt{3}}$,

$tan\phi = etan\theta = \frac{1}{\sqrt{3}}.\sqrt{3} = 1$, so that $\phi = 45°$

And $v = u\sqrt{\frac{1+e^2tan^2\theta}{1+tan^2\theta}} = u\sqrt{\frac{1+tan^2\phi}{1+tan^2\theta}} = u\sqrt{\frac{1+1}{1+3}} = \frac{u}{\sqrt{2}}$

What relation must hold between $tan\theta$ and e, in order for the outgoing path to be perpendicular to the incoming path?

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If $\theta + \phi = 90°$, $tan\phi = etan\theta$ and $tan\phi = \tan(90° - \theta)$

$= cot\theta = \frac{1}{tan\theta}$

Hence $etan\theta = \frac{1}{tan\theta}$, so that $tan^2\theta = \frac{1}{e}$ and $tan\theta = \frac{1}{\sqrt{e}}$

For the same situation, express v in terms of u and e.

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Solution

$$v = u\sqrt{\frac{1+e^2tan^2\theta}{1+tan^2\theta}} = u\sqrt{\frac{1+e^2(\frac{1}{e})}{1+\frac{1}{e}}} = u\sqrt{\frac{e+e^2}{e+1}} = u\sqrt{e}$$

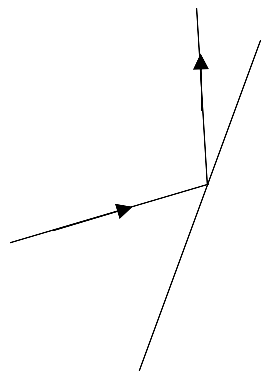
For the same situation, what is the smallest possible value for θ?

[For the same situation, what is the smallest possible value for θ?]

Solution

$tan\theta$, and hence θ, is minimised when e is maximised; ie when $e = 1$ and $tan\theta = 1$, so that $\theta = 45°$

Vector approach, for the general case where the plane (represented by a line) is at an angle to the coordinate axes.



Suppose that the line has direction vector $\begin{pmatrix}2\\5\end{pmatrix}$, and that the incoming velocity is $\begin{pmatrix}4\\1\end{pmatrix}\ ms^{-1}$, with $e = \frac{1}{2}$.

The incoming velocity can be broken down into a vector parallel to the line, and a vector perpendicular to it.

By projecting the incoming velocity onto the line, find the vector parallel to the line.

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Solution

The magnitude of the incoming velocity parallel to the line is:

$\frac{\binom{4}{1}.\binom{2}{5}}{\left|\binom{2}{5}\right|}$, and so the required vector is $\frac{\binom{4}{1}.\binom{2}{5}}{\left|\binom{2}{5}\right|}\frac{\binom{2}{5}}{\left|\binom{2}{5}\right|}$ or $\frac{\binom{4}{1}.\binom{2}{5}}{\left|\binom{2}{5}\right|^2}\binom{2}{5}$

$= \frac{13}{29}\binom{2}{5}$, noting that $\binom{2}{5}$ is pointing in the right direction.

[The direction vector $\binom{-2}{-5}$ would also represent the line in the diagram, but would be opposite to the direction of motion of the ball.]

Find the vector perpendicular to the line.

Find the vector perpendicular to the line.

Solution

A vector perpendicular to the line, in the direction of the ball's motion, is $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$, and so the required vector is $\frac{\begin{pmatrix} 4 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -2 \end{pmatrix}}{\left|\begin{pmatrix} 5 \\ -2 \end{pmatrix}\right|^2} \begin{pmatrix} 5 \\ -2 \end{pmatrix} = \frac{18}{29} \begin{pmatrix} 5 \\ -2 \end{pmatrix}$

Find the outgoing velocity (in vector form).

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Solution

Outgoing velocity is $\frac{13}{29}\begin{pmatrix}2\\5\end{pmatrix} - e\,\frac{18}{29}\begin{pmatrix}5\\-2\end{pmatrix}$

$= \frac{13}{29}\begin{pmatrix}2\\5\end{pmatrix} - \frac{1}{2}.\frac{18}{29}\begin{pmatrix}5\\-2\end{pmatrix}$

$= \frac{1}{29}[\begin{pmatrix}26\\65\end{pmatrix} - \begin{pmatrix}45\\-18\end{pmatrix}]$

$= \frac{1}{29}\begin{pmatrix}-19\\83\end{pmatrix}$ or $\begin{pmatrix}-\frac{19}{29}\\\frac{83}{29}\end{pmatrix}$