Normal Distribution - Q1[12 marks] (10/6/21)

Exam Boards

OCR : AL (Year 2)

MEI: AL (Year 2)

AQA: AL (Year 2)

Edx: AL (Year 2)

Suppose that the heights (in cm) of adult males in the UK are distributed $N(174,49)$.

(i) Assuming that there are 2.5 cm to an inch, what proportion of adult males in the UK are over 6 ft? Give your answer to 1dp.

[3 marks]

(ii) In another country, the heights of adult males are distributed Normally, such that 10% are over 6 ft and 5% are under 5ft. Find the mean and variance of the distribution. Give your answers to 1dp. [9 marks]

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Solution

(i) $6 \text{ ft} = 6 \times 12 \times \frac{5}{2} = 180 \text{ cm}$

If $X \sim N(174, 7^2)$, then $\text{Prob}(X > 180) = \text{Prob}(\frac{X-174}{7} > \frac{180-174}{7})$

[1 mark]

$= \text{Prob}(Z > 0.857) = 1 - 0.8042 = 0.1958$, from tables.

So 19.6% of adult males in the UK are over 6 ft (1dp). [2 marks]

(ii) $5 \text{ ft} = 5 \times 12 \times \frac{5}{2} = 150 \text{ cm}$

Let height, $Y \sim N(\mu, \sigma^2)$. [1 mark]

Then $\text{Prob}\left(Z > \frac{180-\mu}{\sigma}\right) = 0.1$ [1 mark]

and $\text{Prob}\left(Z < \frac{150-\mu}{\sigma}\right) = 0.05$, [1 mark]

so that , from tables, $\frac{180-\mu}{\sigma} = 1.282$ [1 mark]

and $\frac{150-\mu}{\sigma} = -1.645$ [1 mark]

Hence $\frac{180-\mu}{1.282} = \frac{150-\mu}{-1.645} \Rightarrow -296.1 + 1.645\mu = 192.3 - 1.282\mu$

$\Rightarrow \mu = \frac{192.3+296.1}{1.645+1.282} = 166.86$ [2 marks]

and $\sigma = \frac{180-166.86}{1.282} = 10.2496$ and $\sigma^2 = 105.054$ [2 marks]

Thus the mean is 166.9 cm and the variance is 105.1 cm^2 (1 dp).