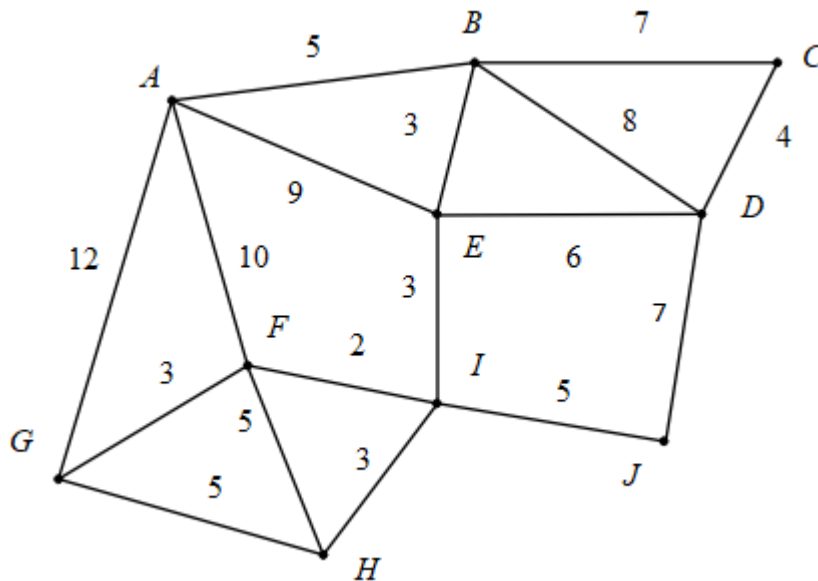


Minimum Connector Exercises (with Sol'ns)

(4 pages; 9/3/26)

(1) For the network below:



(i) Apply Kruskal's algorithm to create a minimum spanning tree (showing the order in which arcs are added), and giving the total weight.

(ii) Apply Prim's algorithm to create a minimum spanning tree - starting at A (showing the order in which nodes are added), and giving the total weight.

(iii) Create a distance matrix for the network.

(iv) Use this matrix to apply Prim's algorithm - starting at J this time.

Solution

(i) FI(2) FG(3) IE(3) BE(3) IH(3) CD(4) IJ(5) AB(5)

[reject FH(5) & GH(5)] ED(6) [spanning tree is complete]

Total weight: 34

(ii) AB(5) BE(5) ED(6) EI(3) IF(2) IH(3) FG(3) DC(4)

Total weight: 34

(iii) & (iv)

	10	6	9	8	5	3	4	7	2	1
	A	B	C	D	E	F	G	H	I	J
A		(5)			9	10	12			
B	5		7	8	(3)					
C		7		(4)						
D		8	4		(6)					
E	9	3		6					(3)	
F	10						3	5	(2)	
G	12					(3)		5		
H						5	5		(3)	
I					3	2		3		(5)
J				7					5	

JI(5) IF(2) FG(3) IE(3) EB(3) IH(3) ED(6) DC(4) BA(5)

Total weight: 34

(2) Minimum connectors M_1 & M_2 have been found for two networks. A new network N is then formed by joining together M_1 & M_2 by the arcs AB and CD , where A & C are nodes in M_1 and B & D are nodes in M_2 .

The tree T is then formed from M_1 and M_2 , together with the shorter of AB and CD . Is T always, sometimes or never a minimum connector for N ?

(2) Minimum connectors M_1 & M_2 have been found for two networks. A new network N is then formed by joining together M_1 & M_2 by the arcs AB and CD , where A & C are nodes in M_1 and B & D are nodes in M_2 .

The tree T is then formed from M_1 and M_2 , together with the shorter of AB and CD . Is T always, sometimes or never a minimum connector for N ?

Solution

If CD (say) is very large (compared with the other arcs in N), then T will be a minimum connector for N , using AB (as one of AB and CD has to be included).

But if AB and CD are both shorter than any of the other arcs in N , then T won't be a minimum connector for N , as one of the arcs of M_1 or M_2 will have been replaced.

So the answer is: sometimes.