

## **Mechanics - Important Ideas: Forces** (16 pages; 12/11/20)

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**(A) Newton's 1st law**

This says that an object continues in motion (in a straight line) with a constant speed (which may be zero), unless acted on by an external force. (It isn't usually needed when answering Mechanics questions.)

**(B) Newton's 2nd law**

This says that  $F = ma$ , where  $F$  is the net force on an object of mass  $m$ , and  $a$  is the acceleration of the object.

It can be considered to be a vector equation, but the simplest approach is usually to resolve the forces in two convenient perpendicular directions, and set up two  $F = ma$  equations.

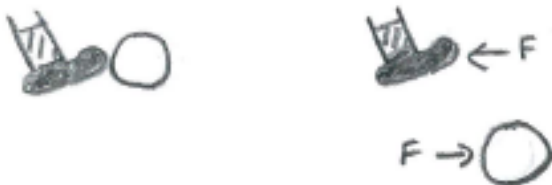
**Note**

As  $F$  may have been defined to be one of several forces acting on the object (friction, for example), it is generally best to write

' $F = ma'$ . Alternatively  $N2L$  can be referred to.

**(C) Newton's 3rd Law**

This says that "if object A exerts a force on object B, then B exerts an equal and opposite (reaction) force on A".

**Example: Football being kicked**

If a football is kicked, then we know from experience that the ball exerts a force on the player's foot. Note though that, despite the apparent symmetry, it is the foot that is the cause of this force.

In this example, the force and reaction force both depend on the nature of the objects in question. Thus a balloon would offer minimal resistance to a football boot, and so the boot would only be able to exert a small force on the balloon.

Don't confuse Newton's 3rd law with equilibrium: In the case of equilibrium, the equal forces being considered act on the same object, whereas in the case of Newton's 3rd law the equal forces act on different objects (in this example, one acts on the ball and the other acts on the foot).

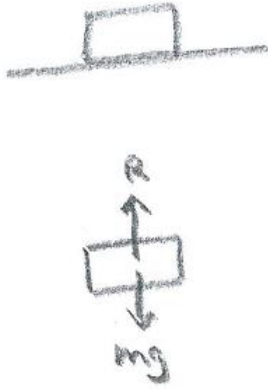
#### **(D) Force Diagrams**

The first step in many Mechanics problems is to create one or more force diagrams. To avoid any confusion, each diagram should show only the forces on the object under consideration (textbooks don't always do this). If not provided in the question, it may be worth having a separate diagram showing the overall situation - including lengths and angles, but omitting forces.

Each force diagram will enable Newton's 2nd law to be applied, and so generate an equation, which will generally enable one unknown variable to be determined.

#### **(E) Reaction force between an object and a surface**

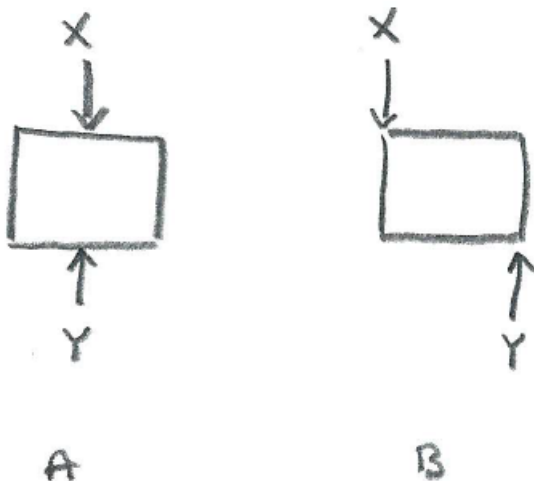
Consider an object of mass  $m$  resting on a horizontal surface. It is tempting to just say that the surface experiences a force of  $mg$  due to the weight of the object. The correct reasoning is as follows:



Drawing a force diagram for the object, and applying Newton's 2nd law gives:  $R = mg$ . Then, by Newton's 3rd law, the reaction force on the surface from the object is  $R$ , which we have found to equal  $mg$ .

(See the "Man in a lift" example below, for a more complicated situation. Also, the note "Important Ideas - Equilibrium" discusses 2 dimensional situations.)

### (F) Particle model



If the forces applied to an object are not liable to cause it to rotate (as in diagram A), then the object may usefully be modelled as a

particle (even if it is quite large). The motion of the object is then determined by applying Newton's 2nd law (in general, in two perpendicular directions).

However, if rotation (or potential rotation) is an issue (as in diagram B), then the theory of moments comes into play. The examples in this note assume that a particle model is appropriate (see the note " Important Ideas - Equilibrium" for non-particle ('rigid body') models).

Where more than one particle is involved (as in most of the following examples), the situation is often referred to as one of "connected particles" (even the man in a lift!)

## (G) Car pulling a trailer

### (G.1) Example

Suppose that a car of mass  $2000\text{kg}$  (2 tonnes) is pulling a trailer of mass  $500\text{kg}$ . The engine of the car supplies a driving force of  $2000\text{N}$ , and there is air resistance of  $600\text{N}$  and  $400\text{N}$  on the car and trailer, respectively. Find the acceleration of the car and trailer, and the tension in the towbar.

### Solution

Considering the car and trailer (and towbar) as a single object, a force diagram can be created, and N2L applied:

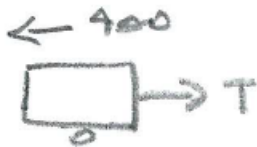


$$2000 - 600 - 400 = (2000 + 500)a$$

$$\Rightarrow a = \frac{1000}{2500} = 0.4 \text{ ms}^{-2}$$

[Although it involves mixing units: a mild acceleration for a car travelling on the motorway at 50 *mph* [miles per hour] would be 1 *mph* per second. Also, the speed of an olympic sprinter (covering 100m in 10 seconds) is  $10 \text{ ms}^{-1}$ , which is approximately  $\frac{10 \times 3600}{1600} = 22.5 \text{ mph}$ . So 1 *mph* is approximately  $\frac{10}{22.5} = 0.44 \text{ ms}^{-1}$ , and 1 *mph* per second is approximately  $0.44 \text{ ms}^{-2}$ .]

Now drawing a force diagram for the trailer:



$$T - 400 = 500(0.4), \text{ so that } T = 600N$$

[The tension in the towbar will be discussed shortly.]

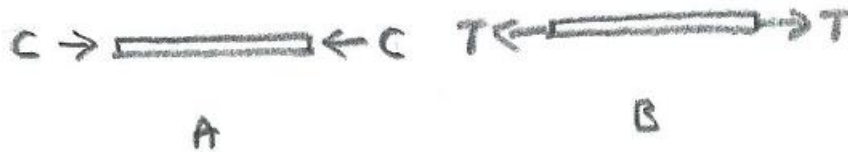
[As a check, we could draw a force diagram for the car:



$$2000 - 600 - T = 2000(0.4), \text{ giving } T = 600N \text{ again.}]$$

## (G.2) Tension in the towbar

Consider a force diagram for the towbar. There are two possibilities: the towbar may be under compression (diagram A), or under tension (diagram B).



Tension will be the more usual situation, with compression generally only occurring in the case of heavy braking (as seen below). For this reason, the symbol  $T$  is invariably used - on the understanding that it may turn out to be negative. (Then, for example, a tension of  $-500\text{ N}$  would be described as a compression of  $500\text{ N}$ .)

The assumption that the towbar is subjected to the same force at each end can be justified as follows:

Suppose that the towbar (under tension) is subject to forces  $T_1$  and  $T_2$  at its two ends, as shown in the diagram.



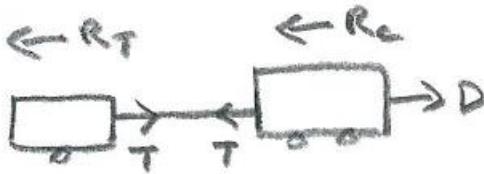
Applying N2L:  $T_1 - T_2 = ma$ ,

where  $m$  is the mass of the towbar, and  $a$  is the acceleration of the car, trailer and towbar.

If the towbar is assumed to be of negligible mass (or 'light'), so that  $m \approx 0$ , then  $T_1 \approx T_2$ .

### (G.3) Textbook diagrams

Textbooks will sometimes (confusingly) include all the forces on a single diagram, and draw the arrows on the towbar pointing away from the car and the trailer. These are intended to be forces on the car and trailer (rather than on the towbar) - see the diagram below.



[To be avoided - some forces are on the car, and some are on the trailer.]

### (G.4) Compression in the towbar

When the car and trailer are decelerating, we can show that the towbar will be under compression if there is heavy braking, or if the resistance on the trailer is sufficiently small:

If the car and trailer are decelerating at a rate  $d$ , then the tension  $T$  in the towbar is given by:

$$R_T - T = m_T d,$$

where  $m_T$  is the mass of the trailer, and  $R_T$  is the resistance on it.

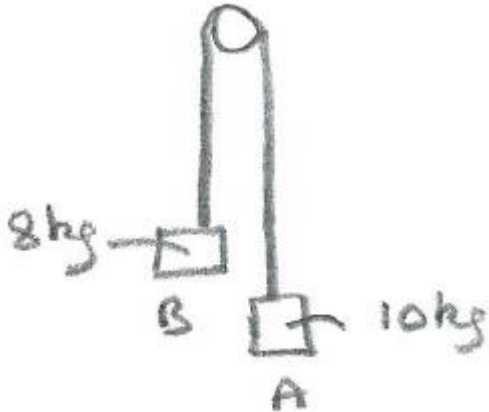
Thus the towbar will be under compression if

$$T = R_T - m_T d < 0; \text{ ie if } R_T < m_T d;$$

ie if either the deceleration is big enough (due to heavy braking), or if  $R_T$  is sufficiently small.

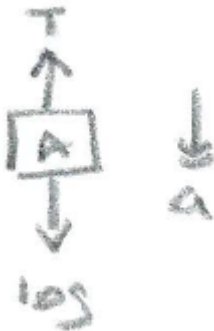
(For large  $R_T$ , the trailer would decelerate at a rate greater than  $d$  if it were uncoupled from the towbar, and the tension is needed in order to obtain the smaller deceleration of  $d$ .)



**(H) Pulley system****Example**

In the case of two blocks on either side of a pulley (as in the diagram), we cannot treat the two blocks and the connecting rope as a single object, as they are not moving in the same direction.

However, we can combine the equations of motion for the two blocks as follows:

**Block A**

$$10g - T = 10a$$

## Block B



$$T - 8g = 8a$$

Adding these two eq'ns gives  $2g = 18a$ , so that  $a = \frac{g}{9} \text{ ms}^{-2}$

[This is a fairly typical fraction of  $g$  to have as an acceleration, and this fact can be used as a check. Compare  $\frac{g}{9} = 1.09 \text{ ms}^{-2}$  with the acceleration of the car which was  $0.4 \text{ ms}^{-2}$ .]

## Notes

(i) The pulley will usually be described as 'smooth', to indicate that there is no frictional force acting on the connecting rope. This will ensure that the tension is the same at both ends of the rope. This can be demonstrated as follows:

In general, a rope (of mass  $m$ ) over a pulley has 3 external forces on it: the tension at the two ends ( $T_1$  &  $T_2$ ) and the frictional force ( $F$ ) due to the pulley.

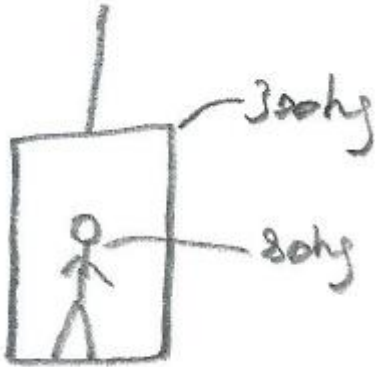
Applying Newton's 2nd law to the rope,

$$T_1 - T_2 - F = ma \text{ (where } a \text{ is the acceleration of the rope)}$$

If the rope has negligible mass, and if  $F$  is also negligible, then  $T_1 \approx T_2$ .

(ii) The rope will usually be described as inextensible, in order to ensure that all components of the system have the same acceleration.

### (I) Man in a lift



**Example 1:** The lift is accelerating upwards at  $0.5 \text{ ms}^{-2}$

Find the tension in the lift cable and the reaction of the man on the floor of the lift.

### Solution

Treating the lift and the man as a single object, we can draw a force diagram and apply N2L:



$$T - (300 + 80)g = (300 + 80)(0.5)$$

$$\Rightarrow T = 380(10.3) = 3914 \text{ N}$$

Then drawing a force diagram for the man:



$$R - 80g = 80(0.5) \Rightarrow R = 80(10.3) = 824 \text{ N}$$

This is the reaction of the floor of the lift on the man, but by N3L the reaction of the man on the floor of the lift is also 824 N.

### Notes

(i) For exam purposes, any symbols used in the force diagrams should be defined.

(ii) The reaction of the floor of the lift on the man is what the man perceives to be his weight. Compare this with his weight when not accelerating with the lift:  $80g = 784 \text{ N}$ . Thus, when the lift is accelerating upwards, the man feels heavier, compared to his natural weight.

**Example 2:** As before, but with the lift accelerating downwards at  $0.5 \text{ ms}^{-2}$

### Solution

For the lift and the man:  $T - (300 + 80)g = (300 + 80)(-0.5)$

$$\Rightarrow T = 380(9.3) = 3534 \text{ N}$$

For the man:  $R - 80g = 80(-0.5) \Rightarrow R = 80(9.3) = 744 \text{ N}$

### Note

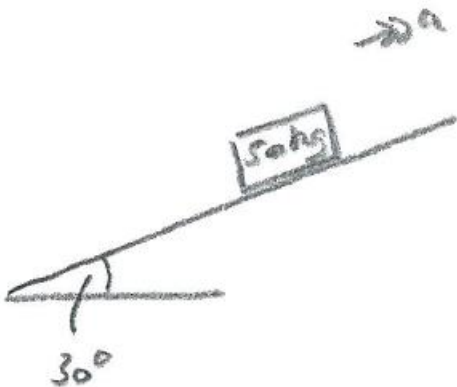
Thus, when the lift is accelerating downwards, the man feels lighter, compared to his natural weight. In the hypothetical case of the lift cable breaking, so that  $a = g$ ,  $R$  becomes zero (ie the man experiences weightlessness).

## (J) Block on a slope

### Example

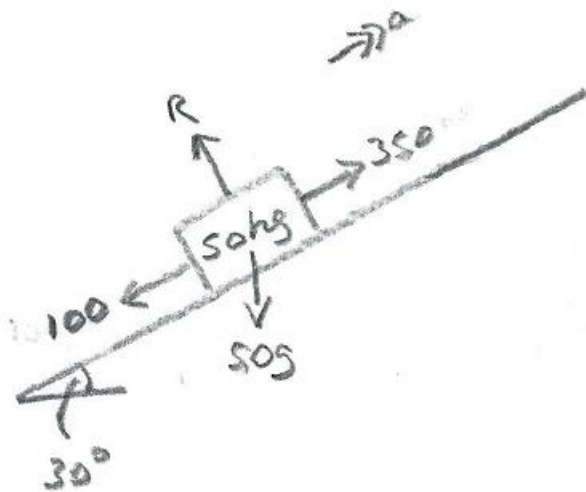
A block of mass  $50 \text{ kg}$  is pulled up a slope (which makes an angle of  $30^\circ$  with the horizontal) by a force of  $350 \text{ N}$ , and there is a frictional force of  $100 \text{ N}$  acting down the slope. Find the acceleration of the block, and the reaction of the slope on the block.

### Solution



This is a two-dimensional situation, and we need to resolve the various forces in two perpendicular directions. For many situations, the most convenient directions to choose are vertically and horizontally. However, in this case the best directions will be along and perpendicular to the slope. This takes advantage of the fact that there will be no acceleration perpendicular to the slope - and this will be true whether or not the block is accelerating. (If the block is either stationary or not accelerating, then forces could in theory be resolved vertically and horizontally, but this is not usually done.)

We can draw a force diagram for the block:



Resolving parallel to the slope and applying N2L:

$$350 - 100 - 50g \sin 30^\circ = 50a$$

$$\Rightarrow a = \frac{1}{50}(250 - 245) = 0.1 \text{ ms}^{-2}$$

Resolving perpendicular to the slope and applying N2L:

$$R - 50g \cos 30^\circ = 0$$

$$\Rightarrow R = 424 \text{ N (3sf)}$$

## (K) Friction

(K.1) Friction between an object and a surface acts to oppose motion or attempted motion of the object.

When an object is stationary, the frictional force will match any applied force (along the surface), up to a certain point. For a particular object on a particular surface this point is determined by the coefficient of friction,  $\mu$ .

According to Coulomb's experimental law,  $F \leq \mu R$ ,

where  $F$  is the frictional force, and  $R$  is the normal (ie perpendicular) reaction of the surface on the object.

Thus, when a force  $X \leq \mu R$  is applied to a stationary object,

$F = X$ , and the object remains stationary.

When  $X > \mu R$ ,  $F = \mu R$ , and by N2L:  $X - \mu R = ma$ ,

where  $m$  is the mass of the object, and  $a$  is its acceleration.

### Notes

(i) In practice, the value of  $\mu$  may reduce significantly once the object is moving. In that case,  $F \leq \mu_s R$  when the object is stationary ( $\mu_s$  being the 'static' coefficient of friction), and

$F = \mu_k R$  once the object is moving ( $\mu_k$  being the 'kinetic' (or 'dynamic') coefficient of friction). To keep things simple, no distinction is generally made between the two types of friction for the purpose of A Level exams.

(ii) Typical values of  $\mu$

steel on steel:  $\mu_s = 0.7$ ,  $\mu_k = 0.6$

wood on wood:  $\mu_s = 0.4$ ,  $\mu_k = 0.2$

ice on ice:  $\mu_s = 0.1$ ,  $\mu_k = 0.03$

(K.2) If an object is at rest on a slope, the frictional force could be acting up or down the slope. An applied force may be just sufficient to stop the object from sliding down the slope, in which case friction will be opposing the attempted motion, which is down the slope, so that the frictional force is up the slope - aiding the applied force. If instead the applied force is not quite enough to move the object up the slope, then the attempted motion is up the slope, and the frictional force is down the slope - countering the applied force).

(K.3) Example

For the example in (J), the frictional force is  $\mu R$ ,

so that  $100 = 50g\cos 30^\circ\mu$ ,

and hence  $\mu = \frac{100}{50(9.8)\left(\frac{\sqrt{3}}{2}\right)} = 0.236$  (3sf)