

## Mechanics - Important Ideas: Collisions (3 pages; 25/10/20)

(1) A perfectly 'elastic' object (where  $e = 1$ ) is one that, on impact with a particular surface, converts all its kinetic energy into elastic potential energy, which is then converted back into the original amount of kinetic energy; ie kinetic energy is conserved.

$e$  is a measure of the relative bounciness of the two materials involved in the collision: the bigger  $e$  is, the more the objects will bounce off each other

(2) Whilst energy is generally lost in a collision (as heat & sound), momentum is conserved because total change in momentum = total impulse = 0, since the objects (A and B, say) exert equal and opposite impulses, by Newton's 3<sup>rd</sup> Law:  $F_A = -F_B \Rightarrow F_A t = -F_B t$

(3) It might seem strange that there is no reference to mass in the Newton's Law of Impact:  $e = \frac{v_s}{v_a}$ . However, it is involved indirectly, since  $v_s$  will be determined by Conservation of Momentum.

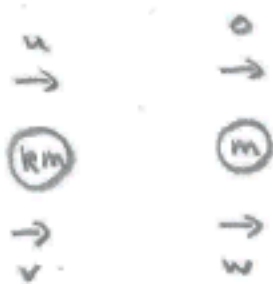
(4) The coefficient of restitution between a ball and a surface can be measured as follows (where  $h_1$  is the height that the ball is dropped from, and  $h_2$  is the height that it rises to):

$$e^2 = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mu^2} = \frac{mgh_2}{mgh_1} \quad (\text{by Conservation of Mechanical Energy})$$

$$= \frac{h_2}{h_1}$$

(5) Conditions for A to reverse its direction when A and B collide (A & B have masses  $km$  &  $m$ ) [See separate note for details.]

(I) A has speed  $u$  and B is stationary



(a) It can be shown that the direction of A is reversed when  $e > k$  (whatever value  $u$  has).

(b) So if  $k \geq 1$ , a change of direction isn't possible.

(c) If  $k < 1$ , a change of direction will be possible provided  $e$  is sufficiently big. Note that a bigger  $e$  means that A and B bounce off each other more.

(II) A and B are moving in the same direction; A has speed  $\lambda u$  ( $\lambda > 1$ ) and B has speed  $u$ .

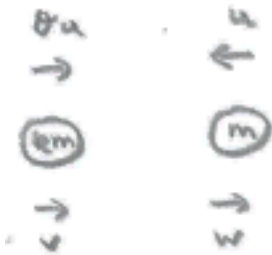


(a) It can be shown that A will never reverse direction if  $\frac{\lambda k + 1}{\lambda - 1} \geq 1$

(b) In particular, A will never reverse direction if  $k \geq 1$  or  $\lambda \leq 2$

(c) If  $k < 1$  &  $\lambda > 2$ , then A will reverse direction if certain further conditions apply to  $k$  &  $\lambda$ , provided that  $e$  is big enough.

(III) A and B are moving in opposite directions; A has speed  $\theta u$  and B has speed  $u$ .



(a) It can be shown that the condition  $k < \frac{\theta+2}{\theta}$  (and sufficiently big  $e$ ) is necessary and sufficient for A to reverse direction.

(b) In particular, if  $k \leq 1$ , then A will reverse direction, for sufficiently big  $e$ .

(c) And if  $k \geq 3$ , then A will never reverse direction.